Chapter 15: The Gravitational Representation of Faraday's Law

Electrodynamics and the mechanics of gravity are very similar, but there are slight differences. This chapter will explore these differences, which lead to interesting results in the mechanics of gravity that differ from those in electrodynamics. Specifically, energy is no longer conserved under gravity, thus supporting the existence of perpetual motion machines. Furthermore, this result could potentially answer the unsolved question of why Saturn's rings exist, a problem that has persisted since Galileo's time.

This chapter will also be structured in the same way: the first half will discuss electrodynamics, and the second half will apply it to the mechanics of gravity.

15.1 Electrostatic Polarization

When an electric charge is brought near an uncharged conductor, an imbalance of positive and negative charges occurs within the conductor. This phenomenon is called electrostatic polarization. The reason this phenomenon occurs is that, macroscopically, an uncharged conductor has an equal number of positive and negative charges that cancel each other out in the electric field, so the conductor appears to have no charge overall. However, when an external charge is brought near this conductor, the newly added Coulomb force causes an imbalance in the conductor's charge.

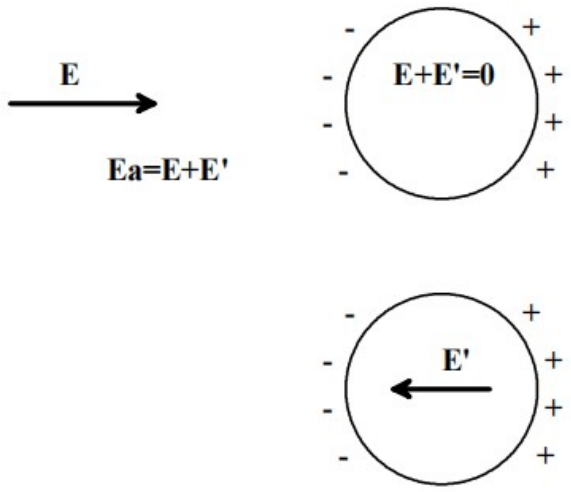
An imbalance in charge can also be said to be the generation of an electric current in the conductor. When the movement of the approaching charge is stopped, the current in the conductor disappears, and the polarization is maintained. Polarized charges create an electric field. Let's call this electric field the polarization electric field $\mathbf{E}'$. If the electric field applied from the outside is $\mathbf{E}$, then the total electric field $\mathbf{E}_a$ is the sum of $\mathbf{E}$ and $\mathbf{E}'$, according to the superposition principle. That is,

$$\mathbf{E}_a = \mathbf{E} + \mathbf{E}' \qquad (15.1)$$

Therefore, since there is no movement of charge inside a conductor, $\mathbf{E} + \mathbf{E}' = 0$,

$$\mathbf{E} = -\mathbf{E}' \qquad (15.2)$$

Thus, the electric field and the polarization electric field are of equal magnitude but opposite in direction. Inside the conductor, the fields **E** and **E**' cancel each other out, resulting in a resultant electric field of zero.



The field created by the polarization electric field of a conductor placed in an externally applied electric field is equivalent to the superposition of the electric field created by the polarization electric field of a dipole.

The electric field created by a polarization electric field is the same as the electric field created by a dipole.

Figure 15.1 Polarization

15.2 Magnetic Induction

When a substance is placed in a magnetic field, magnetic poles appear at both ends of the substance. This phenomenon is called magnetic induction. At this time, it is thought that a magnetic field $\mathbf{B}'$ opposite to the externally applied magnetic field $\mathbf{B}$ is generated inside the substance, and the overall magnetic field $\mathbf{B}_a$ is given as the sum of these magnetic fields. That is,

$$\mathbf{B}_a = \mathbf{B} + \mathbf{B}' \qquad (15.3)$$

Therefore, since magnetic fields are thought to be generated by electric currents, the fact that an induced magnetic field $\mathbf{B}'$ is generated inside a substance placed in a magnetic field means that an electric current is generated inside the substance. Since $\mathbf{B}'$ and $\mathbf{B}$ are in opposite directions, $\mathbf{B}'$ acts to weaken $\mathbf{B}$. If the substance placed in the magnetic field is a conductor, and the substance is not magnetized and $\mathbf{B}$ does not change, then inside the conductor, $\mathbf{B}_a = 0$, that is

$$\mathbf{B} = -\mathbf{B}' \qquad (15.4)$$

This must be the case. If $\mathbf{B}_a$ + 0, then there will be an electric current inside the conductor, and if the conductor is placed in a magnetic field, there will be an uncancelled electric current within the conductor, and the conductor will have an electromotive force.

This phenomenon closely resembles electrostatic polarization. Since a magnetic field is generated by an electric current, and an electric current is generated by an electric field, the fact that an induced magnetic field is generated inside a substance placed in a magnetic field means that there was an electric field inside the substance. Since the things that generate electric fields are due to the fundamental properties of electric charge given by Coulomb's

law, it can be considered that the phenomenon of magnetic induction also arises from the fundamental properties of electric charge.

15.3 Magnetic Flux

Just as we considered electric flux in an electric field, let's consider the amount of magnetic field passing through an arbitrary surface. The amount of magnetic flux density passing through an arbitrary surface is given by its surface integral.

$$\varphi = \int_S \mathbf{B} \cdot d\mathbf{S} \qquad (15.5)$$

This can be written as. This φ is called magnetic flux.

If this surface is a closed surface that contains the source of magnetic field generation,

$$\int_S \mathbf{B} \cdot d\mathbf{S} = 0 \qquad (15.6)$$

However, S is a closed surface

Therefore, the total amount of magnetic flux is zero. This is because there are no magnetic charges equivalent to electric charges in a magnetic field, and therefore there are no emergence points like electric charges.

Gauss's divergence theorem

$$\int_S \mathbf{B} \cdot d\mathbf{S} = \int_V \nabla \cdot \mathbf{B} dv$$

Therefore, equation (15.6) is,

$$\int_V \nabla \cdot \mathbf{B} dv = 0$$

Therefore,

$$\nabla \cdot \mathbf{B} = 0 \qquad (15.7)$$

This can be written as. This equation shows that there are no springs in a magnetic field.

15.4 Faraday's Law

Suppose we have two circuits, A and B, placed close to each other. Circuit A is connected to an ammeter, and circuit B is connected to a battery and a switch. When we turn on the switch in circuit B and allow current to flow, we can observe a deflection in the ammeter of circuit A, but this deflection is instantaneous and stops immediately. Next, when we turn off the switch in circuit B, the ammeter of circuit A deflects instantaneously, but the deflection is the opposite of when the switch was turned on. In this way, when the current in circuit B is turned on and off, current flows in circuit A, which is nearby, so we can consider that there was an instantaneous electromotive force in circuit A. The magnitude of this electromotive force is proportional to the magnitude of the electromotive force of the battery in circuit B, and if a larger current flows through circuit B, we can observe a larger deflection in the ammeter of circuit A.

Furthermore, even if you leave switch B on and move it closer to or further away from A, you can observe the deflection of the ammeter on A. The same thing can be observed if you use a permanent magnet instead of B.

From such experiments, we can conclude that "when the magnetic flux linked to a circuit changes over time, an electromotive force proportional to the change in magnetic flux is generated in that circuit." This phenomenon was discovered by Faraday and is known as Faraday's law or the law of electromagnetic induction.

Furthermore, it can be said that "the electromotive force generated by electromagnetic induction is generated in a direction that produces an electric current that opposes the change in magnetic flux." This way of expressing it is called Lenz's Law.

If we express this phenomenon using a mathematical formula,

$$e = -\frac{d\emptyset}{dt} \qquad (15.8)$$

However, e is the electromotive force generated in the circuit, and ∅ is the magnetic flux.

It can be written as. The negative sign is an expression of Lenz's Law.

This equation is derived from equation (15.5),

$$e = -\frac{d}{dt}\int_S \boldsymbol{B} \cdot d\boldsymbol{S} \qquad (15.9)$$

It can be rewritten as. The sum of the electric fields **E** along the closed circuit c is the integral of the circuit around the tangent line.

$$\oint_c \boldsymbol{E} \cdot d\boldsymbol{s}$$

It can be written as such, and the electromotive force e can be considered to be generated by this electromotive force,

$$e = \oint_c \boldsymbol{E} \cdot d\boldsymbol{s} \qquad (15.10)$$

It can be written as. From equations (15.9) and (15.10),

$$\oint_c \boldsymbol{E} \cdot d\boldsymbol{s} = -\frac{d}{dt}\int_S \boldsymbol{B} \cdot d\boldsymbol{S} \qquad (15.11)$$

This equation is an integral form of Faraday's law.

Stokes' theorem

$$\oint_c \boldsymbol{E} \cdot d\boldsymbol{s} = \int_S \boldsymbol{\nabla} \times \boldsymbol{E} \cdot d\boldsymbol{S}$$

Using this, and $\frac{d}{dt}$ rewriting to $\frac{\partial}{\partial t}$,

$$\int_S \nabla \times \boldsymbol{E} \cdot d\boldsymbol{S} = -\int_S \frac{\partial \boldsymbol{B}}{\partial t} \cdot d\boldsymbol{S}$$

It can be written as. If we rearrange the terms,

$$\int_S (\nabla \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t}) \cdot d\boldsymbol{S} = \boldsymbol{0}$$

And so,

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \tag{15.12}$$

This equation is a differential form of Faraday's law.

In this equation, if the magnetic flux density $\mathbf{B}$ does not change over time,

$$\nabla \times \mathbf{E} = 0 \tag{15.13}$$

Therefore, no electromotive force is generated, but this does not mean that there is absolutely no current flowing through the circuit. This state is the same as the magnetic induction state described in 15.2, and an induced magnetic field is generated in the circuit. This induced magnetic field and the externally applied magnetic field $\mathbf{B}$ cancel each other out, and the magnetic field in the circuit disappears. In the steady state, if we write the externally applied magnetic field in the circuit as $\mathbf{B}$ and the induced magnetic field as $\mathbf{B}^{\prime}$, then their sum is zero,

$$\mathbf{B}' = -\mathbf{B}$$

Therefore, it can be considered that no electromotive force is generated. In other words, equation (15.13) is,

$$\nabla \times \mathbf{E} = -\mathbf{B} - \mathbf{B}'$$

It can be written formally as. Using this term, equation (15.12) becomes,

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} = -\boldsymbol{B} - \boldsymbol{B}' - \frac{\partial \boldsymbol{B}}{\partial t} \tag{15.14}$$

It can be written as.

15.5 Vector Potential

(1) Definition of vector potential

In a magnetic field,

$$\nabla \cdot \mathbf{B} = 0$$

The following relationship always holds. According to the mathematical theorem, equation (11.60), in this case $\mathbf{B}$ is the curl of another vector. If we write this vector as $\mathbf{A}$,

$$\mathbf{B} = \nabla \times \mathbf{A} \qquad (15.15)$$

This can be written as. This vector **A** is called the vector potential. If we take the div of both sides of this equation and check,

$$\nabla \cdot \mathbf{B} = \nabla \cdot (\nabla \times \mathbf{A}) = 0$$

Therefore, it is possible to express equation (15.7) in the form of equation (15.15). This vector **A** is not uniquely determined in general. For example, if $\nabla\varphi$ is a conservative vector and C is a constant vector, and **A** is set as follows,

$$\mathbf{A} = \mathbf{A}_0 + \nabla\varphi + \mathbf{C}$$

Substituting this into equation (15.15)

$$\mathbf{B} = \nabla \times (\mathbf{A}_0 + \nabla\varphi + \mathbf{C}) = \nabla \times \mathbf{A}_0 + \nabla \times \nabla\varphi + \nabla \times \mathbf{C}$$

$$= \nabla \times \mathbf{A}_0$$

Therefore, $\mathbf{A}_0$ is also a vector potential. In this way, there are infinitely many possible values for a vector potential. If there are infinitely many possible values, then calculating something using a vector potential will result in infinitely many results, so the results are not very meaningful. Therefore, we will give conditions to the possible values of a vector potential. This is a matter of selection and definition, and we can give appropriate conditions. In general, a vector potential has,

$$\nabla \cdot \mathbf{A} = 0 \qquad (15.16)$$

The above conditions are given. Ampère's law is,

$$\nabla \times \boldsymbol{B} = \frac{\boldsymbol{j}}{c^2 \varepsilon_0}$$

Therefore, substituting $\mathbf{B} = \nabla \times \mathbf{A}$ into $\nabla \times \mathbf{B}$ on the left side of this equation,

$$\nabla \times \mathbf{B} = \nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

Therefore, from condition (15.16),

$$\nabla \times \nabla \times \mathbf{A} = -\nabla^2 \mathbf{A}$$

Therefore, Ampère's law is,

$$\nabla^2 \boldsymbol{A} = -\frac{j}{c^2 \varepsilon_0} \qquad (15.17)$$

This can be written as. If we write this equation by separating it into its components,

$$\nabla^2 A_x = -\frac{j_x}{c^2 \varepsilon_0}$$
$$\nabla^2 A_y = -\frac{j_y}{c^2 \varepsilon_0}$$
$$\nabla^2 A_z = -\frac{j_z}{c^2 \varepsilon_0}$$

These equations are Poisson's equations.

$$\nabla^2 \varphi = -\frac{\rho}{\varepsilon_0}$$

It is isomorphic to the previous equation and can be solved.

(2) The physical meaning of vector potential

The concept of a magnetic field is itself quite difficult to grasp physically, but what exactly is the physical meaning of this vector potential, which is a vector with a curl applied to it that becomes a magnetic field?

In the mathematical derivation of the magnetic field in 14.5,

$$\mathbf{X} = \mathbf{v} \times \mathbf{E}$$
$$\mathbf{X} = \mathrm{c}^2 \mathbf{B}$$

We have shown that by setting this and performing calculations, we can derive Ampère's law. From these two equations,

$$\mathrm{c}^2 \mathbf{B} = \mathbf{v} \times \mathbf{E} \qquad (15.18)$$

It can be written as. The electric field **E**

$$\mathbf{E} = -\nabla\varphi$$

If we set this and substitute it into equation (15.18), and note that changing the order of the vector product results in a negative value,

$$c^2\mathbf{B} = \nabla\varphi \times \mathbf{v}$$

This is the result. If you take rotate both sides,

$$c^2 \nabla \times \mathbf{B} = \nabla \times (\nabla\varphi \times \mathbf{v})$$

This is the result. Here, according to the vector operation formula (11.55),

$$\nabla \times (\varphi\mathbf{v}) = \nabla\varphi \times \mathbf{v} + \varphi(\nabla \times \mathbf{v})$$

And, vector **v** if we assume a constant velocity, then from equation (11.52), $\nabla \times \mathbf{v} = 0$. Therefore,

$$\nabla \times (\varphi\mathbf{v}) = \nabla\varphi \times \mathbf{v}$$

It can be written as. If we take the rot operator on both sides of this equation,

$$\nabla \times \nabla \times (\varphi\mathbf{v}) = \nabla \times (\nabla\varphi \times \mathbf{v})$$

Therefore, if we substitute this equation into equation (15.19),

$$c^2 \nabla \times \mathbf{B} = \nabla \times \nabla \times (\varphi \mathbf{v})$$

Therefore, if we substitute the relationship $\mathbf{B} = \nabla \times \mathbf{A}$,

$$\nabla \times \nabla \times \mathbf{A} = \nabla \times \nabla \times (\frac{\varphi v}{c^2})$$

In other words, the vector potential is

$$\boldsymbol{A} = \frac{\varphi v}{c^2} + \nabla \gamma + \boldsymbol{C} \qquad (15.20)$$

However, γ is an arbitrary scalar function and $\mathbf{C}$ is a constant vector.

It can be written. If we set γ = 0 and $\mathbf{C}$ = 0,

$$\boldsymbol{A} = \frac{\varphi v}{c^2} \qquad (15.21)$$

This represents a single vector potential. Written in this way, we can see that the vector potential is proportional to the product of the potential and its motion. In other words, if there is a conservative field such as an electric field and that field is in motion, then the motion of the potential (vector potential) arises in the surrounding space.

15.6 Alternative expressions of the stationary form of Faraday's Law

Let's rewrite Ampère's Law and consider another expression of Faraday's Law in a stationary form.

Ampère's Law is,

$$\nabla \times \boldsymbol{B} = \frac{\boldsymbol{j}}{\varepsilon_0 c^2}$$

It is written as. If we take rot on both sides,

$$\nabla \times \nabla \times \boldsymbol{B} = \frac{1}{\varepsilon_0 c^2} \nabla \times \boldsymbol{j} \qquad (15.22)$$

Here, the formula for vector operations

$$\nabla \times \nabla \times \mathbf{B} = \nabla(\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B}$$

Using this, since $\nabla \cdot \mathbf{B} = 0$, equation (15.22) is,

$$-\nabla^2 \mathbf{B} = \frac{1}{\varepsilon_0 c^2} \nabla \times \mathbf{j}$$

This is the result. Ohm's Law

$$\mathbf{j} = \delta \mathbf{E}$$

This is an approximate experimental law and cannot be said to generally hold true, but there are cases where it can be considered to hold true. In such cases, we can think of this relationship as being able to substitute,

$$\nabla \times \boldsymbol{E} = -\frac{\varepsilon_0 c^2}{\delta} \nabla^2 \boldsymbol{B} \qquad (15.23)$$

This can be written as. This equation is the equation for a magnetic field due to an electric current in a vacuum. Substitute $\mathbf{B} = \nabla \times \mathbf{A}$ (vector potential) into this equation.Substituting $\mathbf{j} = \delta \mathbf{E}$, we get the well-known equation,

$$\nabla^2 \boldsymbol{A} = \frac{\boldsymbol{j}}{\varepsilon_0 c^2} \qquad \text{again}(15.17)$$

Therefore, we can obtain this. Thus, equation (15.23) is another expression of equation (15.17), and it can be seen that the magnetic field due to a steady current has an electromotive force expressed by equation (15.23) in a vacuum.

On the other hand, if a conductor is placed around this steady magnetic field **B**, the field inside the conductor will be in the steady-state form of Faraday's law.

$$\nabla \times \boldsymbol{E}_{\mathrm{a}} = 0 \qquad (15.24)$$

It is written as. In a vacuum, there was a magnetic field represented by equation (15.23), but it can be seen that there is none in a conductor. The reason why the magnetic field disappears can be explained by considering that this composite electric field $\mathbf{E}_{\mathrm{a}}$ is a combination of the electric field $\mathbf{E}$ and the polarization electric field vector generated by the magnetic field, and that they cancel each other out due to conflicting effects. If we write this polarization electric field vector as $\mathbf{E}'$,

$\mathbf{E}_{\mathrm{a}} = \mathbf{E} + \mathbf{E}'$, and equation (15.24) is,

$$\bigtriangledown \times (\mathbf{E}+\mathbf{E}') = 0 \qquad (15.25)$$

It can be written as. From equations (15.23) and (15.25), the polarization vector due to the magnetic field is,

$$\nabla \times \boldsymbol{E}' = -\frac{\varepsilon_0 c^2}{\delta}\nabla^2 \boldsymbol{B}' = \frac{\varepsilon_0 c^2}{\delta}\nabla^2 \boldsymbol{B} \qquad (15.26)$$

This is how it is expressed. The magnetic field $\mathbf{B}'$ that appears here is an induced magnetic field produced by the polarization current caused by the magnetic field.

Therefore, another representation of the stationary form of the approximate Faraday law, for which Ohm's law can be considered to hold, can be thought of as the group of equations represented by equations (15.23), (15.25), and (15.26).

15.7 Fundamental Differences Between Electricity and Gravity

Up until now, we have discussed electrodynamics, but from now on, we will use this knowledge of electrodynamics to discuss the mechanics of gravity.

Electrodynamics includes phenomena such as electrostatic polarization, and the reason why such phenomena occur is clearly dependent on the distinction between positive and negative charges and the direction of the force, as described by Coulomb's law. In other words, electric charges have the following characteristics:

Charges of the same sign repel each other.

Charges of opposite signs attract each other.

It is thought that the phenomenon of polarization occurs due to these conflicting effects.

What about gravity? The basic characteristics of gravity are:

Objects with mass exert an attractive force on objects with the same sign of mass.

Therefore, there are no conflicting effects like electric charge. In other words, while electric charge has two types of roles, an object with mass has only one type of role. This means that in the mechanics of gravity, we can consider that the effect of polarization does not exist. In fact, such an effect has not been observed.

Furthermore, magnetic induction can essentially be considered another aspect of phenomena like electrostatic polarization. Therefore, it can be argued that magnetic induction does not exist in the mechanics of gravity.

15.8 Rewriting Ampère's Law in a Gravitational Field

In the same way that we rewrote Ampère's law in section 15.6, we can rewrite Ampère's law in a gravitational field. Ampère's law in a gravitational field is:

$$\nabla \times \boldsymbol{B}_g = \frac{\boldsymbol{j}_g}{G_0 c^2}$$

It was written that if you calculate it the same way as 15.6,

$$-\nabla^2 \boldsymbol{B}_g = \frac{1}{G_0 c^2} \nabla \times \boldsymbol{j}_g \qquad (15.27)$$

It can be written as. In a gravitational field, there are cases where it can be written in a form similar to Ohm's law. For example, it is empirically known that if the velocity of an object moving in the air is sufficiently small, the air resistance can be considered to be proportional to the velocity. If the proportionality constant of resistance is k and the velocity is $\mathbf{v}$, then the resistive force $\mathbf{F}$ is

$$\mathbf{F} = \mathrm{km}\mathbf{v}$$

It can be written as. Let the resistance force be $\mathbf{F} = \mathrm{m}\mathbf{E}_\mathrm{g}$ and the mass flow be $\mathbf{j}_\mathrm{g} = \mathrm{m}\mathbf{v}$.

$$\mathrm{m}\mathbf{E}_\mathrm{g} = \mathrm{k}\mathbf{j}_\mathrm{g}$$

It can be written as, and if we set $\delta_g = \frac{m}{k}$,

$$\mathbf{j}_\mathrm{g} = \delta_\mathrm{g}\mathbf{E}_\mathrm{g}$$

It can be written as, and can be written in the same format as Ohm's law.

The positive direction of the gravitational field $\mathbf{E}_g$ is taken from the source of the gravitational field to a point at infinity, and the direction of motion of an object due to that field is opposite, therefore, the field $\mathbf{E}_g$ and the flow The direction of $\mathbf{j}_g$ is opposite to that of electricity. Keep this in mind when determining the sign. Therefore, Ohm's law in a gravitational field is,

$$\mathbf{j}_g = -\delta_g \mathbf{E}_g \qquad (15.28)$$

It can be written as. When such a relationship holds approximately, it can be formally substituted into equation (15.27),

$$\nabla \times \boldsymbol{E}_g = \frac{G_0 c^2}{\delta_g} \nabla^2 \boldsymbol{B}_g \qquad (15.29)$$

It can be written as. In the mechanics of gravity, we can consider that phenomena such as electrostatic polarization and magnetic induction do not exist as in electrodynamics. Therefore, when considering the stationary form of Faraday's law in a gravitational field, the rot of $\mathbf{E}_g$ It is likely that it is not zero.

15.9 Faraday's Law in a Gravitational Field

We formally apply Faraday's Law,

$$\nabla \times \boldsymbol{E} = -\boldsymbol{B} - \boldsymbol{B}' - \frac{\partial \boldsymbol{B}}{\partial t} \qquad \text{again}(15.14)$$

I wrote this. The reason I wrote it this way is that I wanted to try an analogous application to the mechanics of gravity. The fundamental difference between electricity and gravity is the point described in 15.7, and conversely, it can be said that there are no other differences in mathematical formulation. Rewrite using $\mathbf{B} = \bigtriangledown \times \mathbf{A}$,

$$\nabla \times \boldsymbol{E} = -\nabla \times \boldsymbol{A} - \nabla \times \boldsymbol{A}' - \frac{\partial}{\partial t}\nabla \times \boldsymbol{A} \qquad (15.30)$$

Therefore,

$$\boldsymbol{E} = -\nabla\varphi - \boldsymbol{A} - \boldsymbol{A}' - \frac{\partial \boldsymbol{A}}{\partial t} \qquad (15.31)$$

This is one solution to equation (15.30). When this equation is rewritten in the form for a gravitational field, phenomena like magnetic induction do not exist in the case of gravity, so the equivalent of $\mathbf{A}'$ is zero, and the relationship between the directions of the magnetic field and electric field is opposite because charges repel each other with the same sign, whereas in gravity they attract each other with the same sign.

Therefore, The sign of -A and $-\frac{\partial \boldsymbol{A}}{\partial t}$ will be the opposite,

$$\boldsymbol{E}_g = -\nabla\varphi_g + \boldsymbol{A}_g + \frac{\partial \boldsymbol{A}_g}{\partial t} \qquad (15.32)$$

It could be written as. Just as with electrodynamics, in the mechanics of gravity as well...

$$\bigtriangledown \cdot \mathbf{B}_g = 0 \qquad (15.33)$$

The relationship clearly holds, and the vector potential can be defined in the same way as in electrodynamics.

$$\boldsymbol{A}_g = \frac{\varphi_g \boldsymbol{v}}{c^2} \qquad (15.34)$$

This is a vector potential in a gravitational field. Substituting this into equation (15.32),

$$\boldsymbol{E}_g = -\nabla\varphi_g + \frac{\varphi_g \boldsymbol{v}}{c^2} + \frac{\partial \varphi_g \boldsymbol{v}}{c^2 \partial t} \qquad (15.35)$$

It can be written as. If we take the rot operator in equation (15.32),

$$\nabla \times \boldsymbol{E}_g = \boldsymbol{B}_g + \frac{\partial \boldsymbol{B}_g}{\partial t} \qquad (15.36)$$

This can be written as. This equation is considered to be Faraday's law in a gravitational field. The stationary form of this equation is

$$\nabla \times \boldsymbol{E}_g = \boldsymbol{B}_g \qquad (15.37)$$

This is the result. Faraday's law was obtained experimentally, but as mentioned in 15.8, it was predicted that the stationary form of Faraday's law in a gravitational field is not zero when Ampere's law in a gravitational field is rewritten. In other words, theoretically, the stationary form of Faraday's law in a gravitational field is not zero. This equation is thought to correspond to that.

15.10 Forces Affected by a Moving Gravitational Field

If an object with mass is moving at a constant velocity $\mathbf{v}$, then that object creates a moving gravitational field, and if that field is $\mathbf{E}_g$, then from equation (15.35),

$$\boldsymbol{E}_g = -\nabla\varphi_g + \frac{\varphi_g \boldsymbol{v}}{c^2} \qquad (15.38)$$

It can be written as. Since force is obtained by multiplying $\mathbf{E}_g$ by mass, if there is an object of mass m in $\mathbf{E}_g$, the force acting on that object is,

$$\boldsymbol{F} = m\boldsymbol{E}_g = m(-\nabla\varphi_g + \frac{\varphi_g \boldsymbol{v}}{c^2}) = -m\nabla\varphi_g + m\frac{\varphi_g \boldsymbol{v}}{c^2} \tag{15.39}$$

This can be written as. The second term on the right-hand side indicates that there is a potential flow around it, and this flow also exerts a force on the object. For example, a rotating planet will create a rotational potential flow around it. Suppose we throw a meteorite-like object, which is sufficiently light compared to the planet, with an initial velocity $\mathbf{v}_0$towards the center of this planet from a distance so great that the planet's gravity is almost negligible, ideally from infinity. The meteorite is dropped from infinity to prevent it from having initial momentum due to the planet's influence. What kind of trajectory will the meteorite take when it falls to the planet?

If a planet is not rotating, a meteorite will fall straight towards the planet's center. This is because the second term on the right-hand side of equation (15.39) becomes zero in this case. If a term like the second term on the right-hand side of equation (15.39) does not exist, then even if the planet is rotating, a meteorite will fall straight towards the planet's center, and the surrounding field will not change whether the planet is rotating or not. If a term like the second term on the right-hand side of equation (15.39) does not exist, then gravitational magnetic fields and gravitational magnetic forces also do not exist. However, the existence of such concepts has already been confirmed by experiment, and if a rotating gravitational field and a non-rotating gravitational field are the same, then we must conclude that the state of motion does not change between a rotating gyroscope and a non-rotating gyroscope. This is clearly incorrect, and a term like the second term on the right-hand side of equation (15.39) exists, and it is thought that meteorites are dragged along in the direction of the planet's rotation and fall in a spiral orbit.

The second term on the right-hand side of equation (15.39) represents velocity. For an object moving together with the source of the gravitational field, its relative velocity is zero, and no force acts upon it. Therefore, when observed in a coordinate system fixed on Earth, such a term disappears.

Furthermore, using the second term on the right-hand side of equation (15.39), we can explain why Saturn has rings. Saturn is a massive planet, with a diameter about 10 times that of Earth and a mass about 100 times greater, but it rotates about twice as fast as Earth, so one day on Saturn is about half a day on Earth. Such a massive planet creates a huge gravitational field, and because of its high rotational speed, the effect of the second term on the right-hand side is more pronounced. It is clear that the effect of the second term on the right-hand side is greatest near the equator and least near the poles when compared near the planet's surface. For example, if we consider the Earth not to be revolving, a person at the equator would move the equivalent of the Earth's circumference in one day, while a person at the North Pole would hardly move at all. Therefore, the field around a rotating planet is largest near the equator and smallest near the poles, and objects around the planet are concentrated around the equator.

15.11 Energy of a Moving Gravitational Field

The second term on the right-hand side of equation (15.39) also shows that the energy of a moving gravitational field is not conserved. The law of inertia states that if an object is accelerated, it will continue to move forever unless an external force is applied to it. However, a moving object is also a source of a moving gravitational field. The force due to a gravitational field moving at a constant speed is expressed by equation (15.39). The first term on the right-hand side of this equation is a so-called conservative force, but the second term is not. For example, if an object is rotated, it will continue to rotate forever unless an external force is applied to it. However, a rotating gravitational field is formed around the object, and if another object is placed around it, this other object will experience a force related to the direction of rotation as shown in the second term. The integral of such a rotational force over a line distance will not be zero. Therefore, this force is not a conservative force. Furthermore, it cannot be assumed that the source object of a rotating gravitational field loses its rotational energy by exerting a force on surrounding objects. For example, using the analogy of a water flow, if a waterwheel on the surface of water is rotated, the surrounding water begins to move due to the rotational energy of the waterwheel. If another object is brought into this water flow field, that object will begin to move due to the force of the water flow. In this case, the object has been accelerated by the energy of the water flow field, but whether or not this energy of the water flow field is used, the energy required to rotate the waterwheel does not change. In the case of a gravitational field, no energy is required to keep an object rotating. A rotating gravitational field can impart energy to surrounding objects, just like this water flow field. Therefore, this energy is not conserved, and if this force is used, an infinite amount of energy can be extracted.

This shows that the law of conservation of energy does not hold true for a moving gravitational field. Among the various conservation laws that exist in the world of physics, the law of conservation of energy is given a special status, and it is said that with respect to energy, it encompasses all conservation laws. Physicists seem to consider the law of conservation of energy to be a very important physical law, but everyone agrees that this law is not stated for any particular phenomenon, but rather is philosophical. In other words, this law was neither obtained from experimental facts nor from the theoretical construction of existing knowledge. In fact, no one can prove that this law is correct. This is because this law holds true even when including unknown things that physicists do not yet know.

Can this law truly be called a law of physics? At the very least, this law lacks the physical basis to claim that anything that does not follow it is not true physics. Rather, the problem is that in an attempt to create laws of physics that conform to this fictional law, physics, which should reflect the real world, has become a fictional world of physics, and this is evident in the knowledge of physics.

15.12 Gravity-Based Perpetual Energy Engine

As discussed in Chapter 14, the entire gyroscope can be propelled by rotating it within an external gravitational field such as that of the Earth (Figure 14.7). This represents the extraction of energy from a conservative force field. In Chapter 12, we stated that energy can be extracted from the conservative force field of wind by using a windmill like a propeller to locally convert the conservative force field into a non-conservative force field. The gyroscope plays a similar role to this windmill; by rotating the gyroscope, the rotational gravitational field generated by the gyroscope and the Earth's gravitational field are superimposed, creating a local non-conservative force field. The rotational speed of the gyroscope does not decrease if mechanical resistance is ignored. The thrust from this propulsion device would not require external energy, because there is no effect that reduces the rotational speed of the gyroscope. Therefore, energy could be extracted by utilizing this thrust. Experiments have already confirmed that this device can propel objects, and whether or not energy can be extracted depends on psychological factors, such as how free people are from the preconceived notion of the law of conservation of energy.

Furthermore, the cone-shaped gyroscope propulsion system we considered in Chapter 14 could also generate thrust without using energy.

According to the law of conservation of energy, no matter how much you move the internal weight, it should be proven that the object will not be propelled in a specific direction. However, since such a device actually does propel itself, this also demonstrates that the law of conservation of energy does not hold true.

15.13 Interactions in Relative Motion

Let us place two infinitely long linear objects with uniform mass distributions parallel to each other and call them A and B. Initially, A and B are at rest in an inertial coordinate system S, and we assume that only A is moved along the linear axis at a constant velocity v. According to the conclusions drawn in this paper, B will start moving, dragged along in the direction of A's motion.

As I have repeatedly stated, if the motion is as described above, the law of conservation of energy will not hold. Here, let's consider what happens in terms of relative motion.

When observed from a coordinate system S_A that moves with A, B is observed moving in the opposite direction at a velocity v, as seen from this coordinate system. According to the general view of physical physicists, since the choice of coordinates is arbitrary, it is impossible to distinguish whether A or B is moving. The only thing that can be considered is their relative velocity. Considering this, according to the conclusion of this book, A must start moving, dragged along by the motion of B. This means that the velocity of A as seen from S decreases. In the end, the velocities of both A and B as seen from S should both be v/2, and energy should be conserved. Therefore, it may be argued that the conclusion stated in this book is incorrect. We can prove that this idea is wrong, because using inertial velocity, we can say that it is A that is moving, not B. I've already explained this in detail, so I don't think anyone would doubt it, but to make it easier to understand, let me state an analogical fact about fluids.

Let's assume that two infinitely long linear rods, each with a fin capable of generating or receiving wind, are placed parallel to each other, similar to the thought experiment with linear objects, and we will call them A and B. Initially, let's assume that A and B are at rest in an approximate inertial coordinate system S that acknowledges the presence of air. Next, let's assume that by continuously supplying energy from an external source, only A will move along the linear rod at a constant inertial velocity v. At this time, B will be affected by the wind generated by A and will be dragged along in the direction of A's motion. When observed from a coordinate system S_A that moves together with A, we will observe that B is moving in the opposite direction at a velocity v. However, A does not receive wind from B. Therefore, A's velocity does not change due to the influence of B. The only thing that affects A's velocity is air resistance, which is independent of the presence or absence of B.

To create wind, air resistance is necessary, and reducing air resistance reduces wind force. In this respect, wind force and air resistance are proportional, and energy is conserved because of this. On the other hand, the law of inertia states that there is no resistance to maintaining the flow of a gravitational field, which can be likened to a wind of gravity. And if surrounding objects are subjected to a force by this flow of the gravitational field, then energy is no longer conserved with respect to gravity.

15.14 What does it mean for energy to be conserved

As mentioned in section 8.8, energy is required to continuously apply a force to a stationary object. For example, if this force is due to a wind field, the kinetic energy of the air decreases in the wind field, so the overall energy of the wind field decreases. This means that when air particles collide with a stationary object, the kinetic energy of these air particles is added to the stationary object, and the air particles lose their kinetic energy due to the reaction effect.

Energy conservation is due to the effect of reaction. For example, consider an experiment where two spheres collide, assuming there is no reaction. When a moving sphere B collides with a stationary sphere A, B exerts a force on A and transfers its momentum. Normally, due to reaction, B receives a force in the opposite direction and loses momentum, but assuming there is no reaction, B will have the same momentum as before the collision. Here, A has gained the momentum of B, so the total momentum becomes twice the momentum of B, and the energy increases after the collision by the amount of B's momentum compared to before the collision. In this case, energy is not conserved.

There is a way to determine whether a reaction force is exerted on an object being accelerated. If inertial acceleration acts on the accelerating object, a reaction force is exerted; if inertial acceleration does not act on the accelerating object, then no reaction force is considered to have been exerted. If an object is pushed, and there is no reaction force, then the object pushing the object cannot exert a force on the object pushing it. This amount of reaction force is inertial acceleration, so when an object has inertial acceleration, it can be considered that there is a reaction force.

Normally, when you push an object, its inertial acceleration always changes. However, as is readily apparent, when an object is accelerated by gravity, its inertial acceleration does not change. This indicates that there is no reaction force when accelerating an object by gravity. The reaction force reduces the energy applied, but since there is no reaction force in the case of gravity, it can be considered that the effect of gravity does not attenuate even when a force is applied to an object. In fact, it is believed that the effect of gravity does not attenuate even when a force is applied to an object along the way. If it did attenuate, the Earth's gravity would hardly reach its surface.

Essentially, a gravitational field is a force field, and because it can transmit force, it can also transmit energy. An object within a gravitational field is accelerated, and therefore, energy is added to that object. For energy to be conserved, as in a wind field, the field's energy must decrease by the amount the object is accelerated. However, in a gravitational field, the field's energy does not diminish even when an object is accelerated. This indicates that, with respect to gravity, energy is not conserved. A gravitational field is an infinite source of energy.