Chapter 14: The Concept of Magnetic Fields

Magnetism has been known since ancient times. Around the 5th century BC, it was known that stones mined near Magnes in Turkey would orient themselves in a specific direction when suspended. Around the same time, it is said that it was discovered that amber could become electrically charged through friction. For a long time, electricity and magnetism were studied independently as separate phenomena. According to the general theory until the beginning of the 19th century, electricity and magnetism were completely different, and considering their relationship was out of the question. In the winter of 1819-1820, the Danish physicist Erstett first discovered the deflection of a magnetic needle caused by an electric current in a public demonstration experiment. This discovery is considered to have been accidental, but it is well known that this achievement became the starting point for later research in electromagnetism. In this way, it became possible to link electricity and magnetism through experimentation, but even today there is no theory that derives a connection between electricity and magnetism, and a theoretical gap exists between them. In other words, if we ask what magnetism is, and we disregard the existence of magnets, we can experimentally determine that it is something that arises around an electric current. However, if we ask why it arises, no one knows the answer.

In this chapter, we will offer one answer to this question. This answer will be applied as the most important part of our theory of gravity.

14.1 Electric Current

A moving electric charge is called an electric current. As is clear from this definition, a constant electric current $\mathbf{J}$ can be expressed as the product of a charge Q and its velocity $\mathbf{v}$. That is,

$$\mathbf{J} = \mathrm{Q}\mathbf{v} \qquad (14.1)$$

Using charge density ρ, since charge density is the amount of charge per unit volume, the product of charge density and velocity is the current, which is the amount of charge per unit volume. This current is called current density $\mathbf{j}$.

$$\mathbf{j} = \rho \mathbf{v} \tag{14.2}$$

It can be written as.

Generally, electric current is defined as the amount of charge moved per unit time. For example, when a charge dQ passes through the cross-section of a single wire in dt seconds, the current J that passes through that cross-section is:

$$\mathrm{J} = \frac{dQ}{dt}[C/s] \qquad (14.3)$$

The unit quantity of electric current, [coulombs/second], is commonly called the well-known unit quantity [ampere]. Regarding unit quantities,

$$[\mathrm{A}] = [\mathrm{C/s}]$$

It is converted to this. If we write equation (14.3) in integral form,

$$\mathrm{Q} = \int \mathrm{Jdt}$$

This is also clear from the definition.

Current density j can also be thought of as the current passing through a unit area per unit time. For a current moving at a constant speed, the relationship is displacement = velocity × time, and if time is the unit time, then displacement = velocity.

This is because the product of this displacement and the unit area traversed is the volume, and the total current traversed divided by the volume is the current density. Therefore, when a constant current density $\mathbf{j}$ passes perpendicularly through the cross-section of a wire of area S, the total current is the sum over that area.

$$J = \int_S j dS$$

Therefore, in the general case where $\mathbf{j}$ is not perpendicular to the cross-section, it becomes a surface integral, just like Gauss's law.

$$J = \int_S \mathbf{j} \cdot d\mathbf{S} \qquad (14.4)$$

It can be written as: According to Gauss's divergence theorem, for any vector $\mathbf{A}$,

$$\int_S \mathbf{A} \cdot d\mathbf{S} = \int_V \nabla \cdot \mathbf{A} dv$$

Because of this relationship, equation (14.4) is,

$$J = \int_S \mathbf{j} \cdot d\mathbf{S} = \int_V \nabla \cdot \mathbf{j} dv \qquad (14.5)$$

It can also be written as.

If we take a closed surface of any shape with surface area S and volume V in a region where a steady current is flowing, and let ρ be the charge density at any point within volume V, then the total charge Q contained within it is, as described in the previous chapter,

$$Q = \int_V \rho dv$$

And so it was. If charge flows out of a closed surface, the amount per unit time is given by equation (14.3), and by the definition of partial derivative,

If we write $\frac{dQ}{dt}$ as $\frac{\partial Q}{\partial t}$,

$$J = \frac{\partial Q}{\partial t} = \frac{\partial}{\partial t}(\int_V \rho dv) = \int_V \frac{\partial \rho}{\partial t} dv \qquad (14.6)$$

It is possible to write this. Therefore, the amount of charge inside the closed surface decreases by this amount per unit time. That is, the amount is

$$J = -\frac{\partial Q}{\partial t} = -\int_V \frac{\partial \rho}{\partial t} dv \qquad (14.7)$$

Therefore, the amount of charge flowing out of this closed surface can also be expressed by equation (14.4), and since this amount is equal to the amount in equation (14.7),

$$J = \int_S \boldsymbol{j} \cdot d\boldsymbol{S} - \frac{\partial Q}{\partial t} = -\int_V \frac{\partial \rho}{\partial t} dv \qquad (14.8)$$

There is a relationship as. From equation (14.5),

$$\int_V \nabla \cdot \boldsymbol{j} \mathrm{dv} = -\int_V \frac{\partial \rho}{\partial t} dv \qquad (14.9)$$

It can be expressed as. Therefore,

$$\nabla \cdot \boldsymbol{j} + \frac{\partial \rho}{\partial t} = 0 \qquad (14.10)$$

This equation is called the continuity equation.

14.2 Mass Flow

It is possible to define the concept of mass flow in exactly the same way as electric current. Again, since there are many new definitions, I will write them out without omission.

Let's call a moving mass a mass flow. A mass flow can be defined as the mass moved per unit time. For example, when a mass of dM passes through a cross-section in dt seconds, the mass flow $\mathrm{J_g}$ that passes through that cross-section is:

$$J_g = \frac{dM}{dt}[kg/s] \qquad (14.11)$$

It is defined as. If we write equation (14.11) in integral form,

$$\mathrm{M} = \int \mathrm{J_g dt} \qquad (14.12)$$

This is also clear from the definition. Since the quantity known as mass flow is a quantity relating to a moving mass, it can be considered another form of expression for momentum, which is familiar in mechanics. In fact, mass flow is,

$$\mathbf{J}_\mathrm{g} = \mathrm{M}\mathbf{v} \qquad (14.13)$$

It is possible to write it as, and the amount of flow per unit volume is called the flow density. It is called $\mathbf{j}_\mathrm{g}$, and the mass per unit volume is called the mass density ρ_g. If we do that, just like in the case of electric current,

$$\boldsymbol{j}_g = \rho_g \boldsymbol{v} \qquad (14.14)$$

It can be written as.

Just as with electric current, if a constant mass flux density $\mathbf{j}_\mathrm{g}$ passes perpendicularly through a cross-section of area S, the total mass flux can be found by taking the sum over that area.

$$\mathrm{J_g} = \int_\mathrm{S} \mathrm{j_g dS}$$

Therefore, in the general case where $\mathbf{j}_g$ is not perpendicular to the cross section, it becomes a surface integral, just like Gauss's law.

$$J_g = \int_S \mathbf{j}_g \cdot d\mathbf{S} \qquad (14.15)$$

It can be written as. According to Gauss's divergence theorem, equation (14.15) is,

$$J_g = \int_S \mathbf{j}_g \cdot d\mathbf{S} = \int_V \nabla \cdot \mathbf{j}_g dv \qquad (14.16)$$

It can also be written as.

If we take a closed surface of any shape with surface area S and volume V in a region where a steady mass flow is occurring, and let ρ_g be the mass density at any point within volume V, then the total mass M contained within it is, as in the case of electric charge,

$$M = \int_V \rho_g \, dv$$

Therefore, if mass flows out of a closed surface, the amount per unit time is given by equation (14.11), and by the definition of partial derivatives,

$$J_g = \frac{\partial M}{\partial t} = \frac{\partial}{\partial t}\left(\int_V \rho_g dv\right) = \int_V \frac{\partial \rho_g}{\partial t} dv$$

It is possible to write this. Therefore, the mass within the closed surface decreases by this amount per unit time. That is, the amount is

$$J_g = -\frac{\partial M}{\partial t} = -\int_V \frac{\partial \rho_g}{\partial t} dv \qquad (14.17)$$

Therefore, the mass flowing out of this closed surface can also be expressed by equation (14.15), and since this quantity is equal to the quantity in equation (14.17),

$$J_g = \int_S \boldsymbol{j}_g \cdot d\boldsymbol{S} = -\frac{\partial M}{\partial t} = -\int_V \frac{\partial \rho_g}{\partial t} dv$$

There is a relationship between as. From equation (14.16),

$$\int_V \nabla \cdot \boldsymbol{j}_g dv = -\int_V \frac{\partial \rho_g}{\partial t} dv \tag{14.18}$$

It can be expressed as. Therefore,

$$\nabla \cdot \boldsymbol{j}_g + \frac{\partial \rho_g}{\partial t} = 0 \tag{14.19}$$

This equation is the equation of continuity. The equation of continuity also holds true in the mechanics of gravity.

14.3 Right-Hand Rule

Oerstedt discovered magnetism caused by electric current, and through experiments, he found that the relationship between the direction of the current and the direction of the magnetic needle can be likened to a right-hand screw. If the direction of the current is considered positive from the south pole to the north pole, then the direction of the magnetic needle is the direction in which the screw is turned. This relationship between the direction of the current and the direction of magnetism is called the right-hand rule. The fact that the magnetic needle points to the right is due to defining the direction of the magnetic needle as positive from the south pole to the north pole. If the direction of the magnetic needle is defined as positive from the north pole to the south pole, then the direction of the magnetic needle would be to the left. With such a definition, it would be the left -hand rule, and there is no essential meaning in nature to whether it is to the right or left. However, regardless of whether the magnetic needle points to the right or left, the right-hand rule states that for an electric current flowing in a spatially constant direction, the magnetic field around it is not symmetrical. For example, if we imagine the electric current as a long rod moving through water, and the magnetism as the surrounding water, then if we move the rod in a straight line along its extension, the surrounding water will begin to revolve either to the right or to the left of the direction of the rod's motion.

If magnetism is essentially like this, it is a very mysterious thing and seems to be beyond the realm of human imagination.

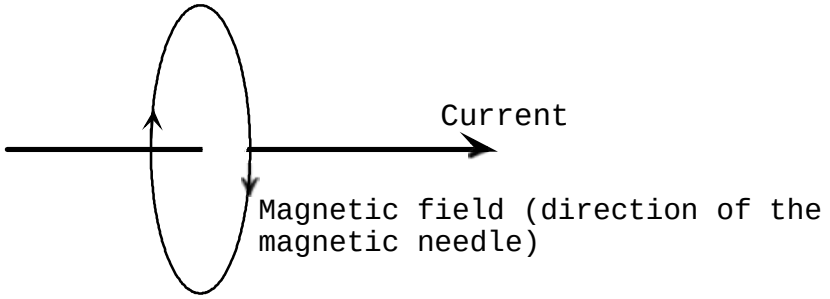


Figure 14.1 Right-hand rulemagnetism

If you actually move a rod in a straight line underwater, the surrounding water will not start to rotate clockwise or counterclockwise around the direction of the rod's motion, but will instead be dragged along in the direction of the rod's motion due to the viscosity of the water. The water moves faster the closer it is to the rod and slower the further away it is. If you place a waterwheel in this water flow, the waterwheel will rotate (Figure 14.2). If we consider a vector that passes through the axis of rotation of this waterwheel, whose magnitude is equal to the rotational speed of the waterwheel, and whose direction is related to the direction of rotation of the waterwheel in a right-hand screw relationship, we can see that this vector is exactly related to the direction of the rod's motion in a right-hand screw relationship. This means that if we consider the motion of the rod as an electric current, the direction of this vector is the same as the direction of a magnetic needle.

While magnetism is generally considered a concept unique to electricity, it is almost impossible to avoid concluding that such a vector is a mathematical quantity that inevitably appears in fluid phenomena such as the flow of water or air. In fact, the water flow experiment mentioned here clearly demonstrates the existence of such a vector, and this vector can be considered a magnetic vector in water flow.

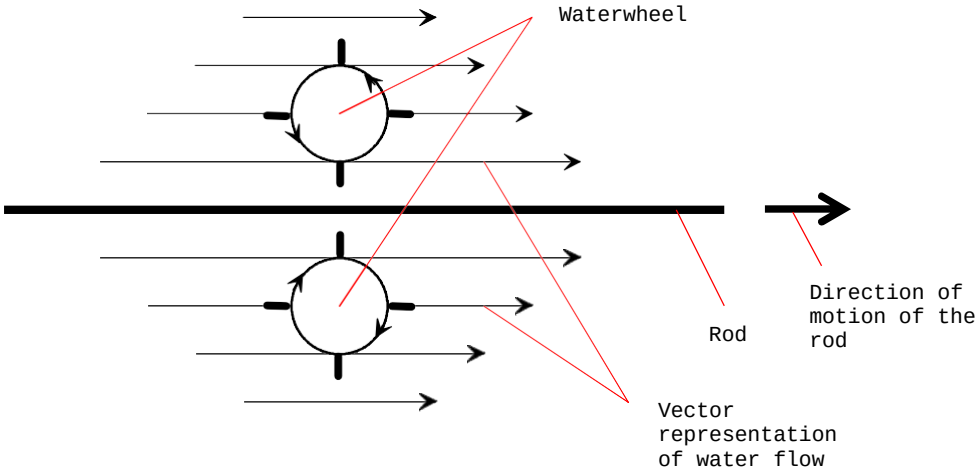


Figure 14.2 The Essence of Magnetic Fields

14.4 Ampère's Law

The right-hand rule qualitatively describes the relationship between steady-state current and magnetism, but the experimental law that includes this rule and provides a quantitative relationship is called Ampère's law. There are various ways to express Ampère's law, but its differential form is...

$$\nabla \times \boldsymbol{H} = \boldsymbol{j} \tag{14.20}$$

$\mathbf{H}$ is magnetic field, $\mathbf{j}$ is current density

It is written as. Alternatively, if we let it be $\mathbf{H} = c^2 \varepsilon_0 \mathbf{B}$,

$$c^2 \nabla \times \boldsymbol{B} = \frac{\boldsymbol{j}}{\varepsilon_0} \tag{14.21}$$

c is speed of light in a vacuum, **B** is magnetic flux density

It is sometimes written as. The physical meaning of this is the right-hand rule described in the previous section, and its mathematical expression is given by equations (14.20) and (14.21). However, if we were to explain it mechanically, it would be better to consider a quantity called angular momentum. Angular momentum is defined as follows:

$$\mathbf{L} = \mathbf{r} \times \mathbf{P} \qquad (14.22)$$

$\mathbf{L}$ is angular momentum, $\mathbf{r}$ is position vector, and $\mathbf{P}$ is momentum.

If we set the starting point of this vector $\mathbf{P}$ to the origin and align its

direction with the z-axis,

$$\mathbf{P} = 0\mathbf{i} + 0\mathbf{j} + |\mathbf{P}|\mathbf{k} = \mathrm{P}\mathbf{k}$$

And if we let **r** = x**i** + y**j** + z**k**, then from equation (6.35),
direction

$$\mathbf{L} = \mathbf{r} \times \mathbf{P} = (\mathrm{x}\mathbf{i} + \mathrm{y}\mathbf{j} + \mathrm{z}\mathbf{k}) \times \mathrm{P}\mathbf{k} = \mathrm{Py}\mathbf{i} - \mathrm{Px}\mathbf{j}$$

If we let $\mathbf{L} = \mathrm{L_x}\mathbf{i} + \mathrm{L_y}\mathbf{j} + \mathrm{L_z}\mathbf{k}$,

$$\mathrm{L_x} = \mathrm{Py}, \mathrm{L_y} = -\mathrm{Px}, \mathrm{L_z} = 0$$

If we take rot for equation(14.22),

$$\nabla \times \boldsymbol{L} = \left(\frac{\partial L_z}{\partial y} - \frac{\partial L_y}{\partial z}\right)\boldsymbol{i} + \left(\frac{\partial L_x}{\partial z} - \frac{\partial L_z}{\partial x}\right)\boldsymbol{j} + \left(\frac{\partial L_y}{\partial x} - \frac{\partial L_x}{\partial y}\right)\boldsymbol{k}$$

$$= 0\mathbf{i} + 0\mathbf{j} + \left(\frac{\partial(-P_x)}{\partial x} - \frac{\partial p_y}{\partial y}\right)\boldsymbol{k}$$

$$= 0\mathbf{i} + 0\mathbf{j} - 2\mathrm{P}\mathbf{k} = -2\mathbf{P}$$

If we set $\mathbf{L} = -2\mathbf{H}_\mathrm{g}$,

$$\nabla \times (-2\mathbf{H}_g) = -2\mathbf{P}$$

$$\nabla \times \mathbf{H}_g = \mathbf{P}$$

This can be written as. This equation is in the same form as equation (14.20). Since the quantity corresponding to momentum in electrodynamics is electric current, it can be seen that the vector -2 times the momentum vector in mechanics corresponds to the quantity corresponding to the rot of the magnetic field in electrodynamics.

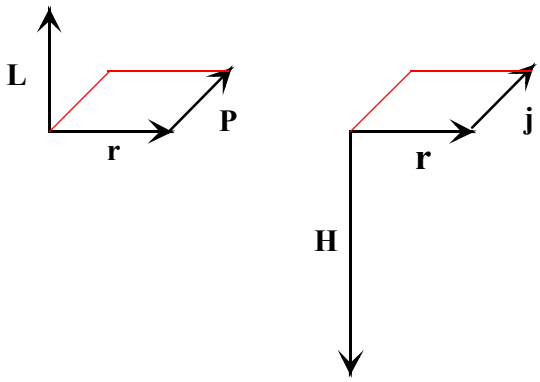


Figure 14.3 Angular Momentum and Magnetic Field

In other words, another form of equation (14.20) is:

$$\mathbf{H} = -\frac{1}{2}\mathbf{r} \times \mathbf{j} \qquad (14.23)$$

Therefore, a magnetic field is a quantity obtained by multiplying the vector product of the electric current and its position vector by $-\frac{1}{2}$, mathematically, can be defined as a magnetic field vector by taking the vector product of any vector and its position vector, and such a magnetic field vector can be defined for any vector.

14.5 Mathematical Derivation of Electrical Magnetic Fields

As confirmed in the previous section, the concept of a magnetic field is not limited to electrical phenomena. From now on, to distinguish it from other types of magnetic fields, we will refer to the magnetic field in electrodynamics as an electrical magnetic field.

Electrical magnetic fields are thought to be generated by the motion of electric charges. In electrodynamics, electrostatics, which does not involve magnetic fields, and electromagnetism, which does involve magnetic fields, are broadly divided into two types. There is no theoretically natural connection between electrostatics and electromagnetism, and the theoretical starting point of electromagnetism is the experiment of Oersted's discovery of magnetism using electric current.

It is clear that magnetic fields are generated by the motion of electric charges, yet it seems unnatural that there is no natural theoretical connection between electrostatics, where the velocity of motion is zero, and electromagnetism, where motion does exist. Understanding this relationship may allow us to theoretically explain the generation of magnetic fields, and it seems that this will allow us to make the laws of electrodynamics more complete. In 14.3, we considered the essence of magnetic fields from a phenomenal perspective, and in 14.4, we stated that a magnetic field vector is simply a matter of definition. Therefore, there is no doubt that if there is a vector like an electric current, a magnetic field vector can be defined. However, while a magnetic field vector has spatial extent, an electric current vector is a local entity flowing through a wire and cannot be considered to have spatial extent. In electrodynamics, the electric field E is what has spatial extent, so it is better in terms of understanding the phenomenon to consider the magnetic field as a quantity relating to the motion of the electric field. The calculations are a bit complicated, but next let's consider the concept of a moving electric field.

In an electrostatic field, the following two equations are obtained.

$$\boldsymbol{E} = -\boldsymbol{\nabla}\boldsymbol{\varphi} \tag{14.24}$$

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0} \tag{14.25}$$

$\mathbf{E}$ is electric field vector, φ is electric potential,

ρ is charge density, ε_0 is permittivity in a vacuum

These equations serve as our starting point. Let $\mathbf{E}$ represent the electrostatic field in a certain coordinate system S', and assume that S' is moving at a constant relative velocity $\mathbf{v}$ as viewed from another coordinate system S. What we will discuss here will not concern ourselves with whether S' and are actually moving or not. What we will discuss here is fundamentally mathematical and does not describe interactions in relative motion, but is the same as asking what a station looks like from the perspective of a train, and is considered to hold true in both cases.

Let $\mathbf{v}\times\mathbf{E}$ be the quantity observed from S to S'. If we let this vector product be **X**,

$$\mathbf{X}=\mathbf{v}\times\mathbf{E} \tag{14.26}$$

Therefore, the necessary and sufficient condition for the existence of the quotient of this vector equation is that $\mathbf{v}\cdot\mathrm{X}=0$ is perpendicular to $\mathbf{v}$ and $\mathbf{X}$. In this case, the quotient is

$$\boldsymbol{E}=\frac{\boldsymbol{X}\times\boldsymbol{v}}{v^2}+k\boldsymbol{v}$$

However, k is an arbitrary constant, $\mathbf{v}^2=\mathbf{v}\cdot\mathbf{v}$

If we set k=0 and ignore this term, equation (14.27) becomes:

$$\mathbf{v}^2\mathbf{E}=\mathbf{X}\times\mathbf{v} \tag{14.28}$$

This is the result. If we take div on both sides of equation (14.28),

$$\nabla \cdot (\mathbf{v}^2\mathbf{E}) = \nabla \cdot (\mathbf{X} \times \mathbf{v}) \qquad (14.29)$$

If we calculate both sides of equation (14.29),

$$\nabla \cdot (\mathbf{v}^2\mathbf{E}) = \nabla \mathbf{v}^2 \cdot \mathbf{E} + \mathbf{v}^2 \nabla \cdot \mathbf{E}$$

Since $\mathbf{v}$ is constant, $\nabla \mathbf{v}^2 = 0$, and therefore

$$\nabla \cdot (\mathbf{v}^2\mathbf{E}) = \mathbf{v}^2 \nabla \cdot \mathbf{E}$$

$$\nabla \cdot (\mathbf{X} \times \mathbf{v}) = \mathbf{v} \cdot \nabla \times \mathbf{X} - \mathbf{X} \cdot \nabla \times \mathbf{v}$$

$\mathbf{v}$ is constant, from equation (11.52)

$\nabla \times \mathbf{v} = 0$, therefore

$$\nabla \cdot (\mathbf{X} \times \mathbf{v}) = \mathbf{v} \cdot \nabla \times \mathbf{X}$$

Therefore, equation (14.29) is,

$$\mathbf{v}^2 \nabla \cdot \mathbf{E} = \mathbf{v} \cdot \nabla \times \mathbf{X} \qquad (14.30)$$

This is the result. Furthermore, since this quotient is $\mathbf{v} \neq 0$, it can be found,

$$\nabla \times \mathbf{X} = \frac{\boldsymbol{v}\, \boldsymbol{v}^2 \nabla \cdot \boldsymbol{E}}{\boldsymbol{v}^2} + \mathbf{k} \times \mathbf{v} \qquad (14.31)$$

However, $\mathbf{k}$ is an arbitrary vector.

This is the result. If we set $\mathbf{k} = 0$ and ignore this term, equation (14.31) becomes,

$$\nabla \times \mathbf{X} = \mathbf{v} \nabla \cdot \mathbf{E} \qquad (14.32)$$

Equation (14.25) also holds true for a moving electric field. Therefore, we can substitute equation (14.25) into equation (14.32),

$$\nabla \times \boldsymbol{X} = \frac{\rho \boldsymbol{v}}{\varepsilon_0} \tag{14.33}$$

Let $\mathbf{X} = c^2\mathbf{B}$, and substitute the current density $\mathbf{j} = \rho\mathbf{v}$, and equation (14.33) becomes:

$$c^2 \nabla \times \boldsymbol{B} = \frac{\boldsymbol{j}}{\varepsilon_0} \tag{14.34}$$

However, c is the speed of light in a vacuum.

This can be written as. This equation is isomorphic to Ampere's law, and from such calculation results, it can be considered that vector $\mathbf{B}$ represents a magnetic field. This suggests that by considering a moving electric field, the relationship between electric current and magnetic field can be mathematically derived.

14.6 Gravitational Magnetic Fields

Besides fluid dynamics and electrical magnetic fields, do such phenomena exist? If magnetic phenomena are a mathematical concept that inevitably appears in phenomena related to fluids, then it can be assumed that such a concept also exists in gravitational fields.

The relationship between magnetic field and current (14.34) obtained in 14.5 was derived from equations (14.24) and (14.25). The equations for the static gravitational field (13.24) and (13.25) are in exactly the same form as equations (14.24) and (14.25). Therefore, by exactly the same calculation,

$$c^2 \nabla \times \boldsymbol{B}_g = \frac{\boldsymbol{j}_g}{G_0} \tag{14.35}$$

However, c is the speed of light in a vacuum,

$\mathbf{B}_g$ is the magnetic vector in a gravitational field,

$\mathbf{j}_g = \rho_g \mathbf{v}$ is density of mass flow

We can obtain as. Here, c is introduced simply for the sake of formality. This equation is the same form as Ampère's law in electrodynamics. Therefore, the vector $\mathbf{B}_g$ represents a magnetic vector in a gravitational field, and we can consider that by considering the motion of a static gravitational field, we have been able to theoretically demonstrate the existence of a magnetic vector. Let us call this vector $\mathbf{B}_g$ the gravitational magnetic field, and let us call this equation (14.35) Ampère's law in a gravitational field.

14.7 Hydrodynamic Magnetic Fields

The discussions up to the previous section confirmed that the concept of a magnetic field is something mathematical, and that similar concepts exist in phenomena other than electricity. However, the important phenomenon in electrodynamics is not the magnetic field, but magnetism, and the question is whether magnetic phenomena exist in phenomena other than electricity.

If current is passed through two parallel wires in the same direction, it can be observed that the two wires attract each other, and if current is passed through them in opposite directions, they repel each other (Figure 14.4).

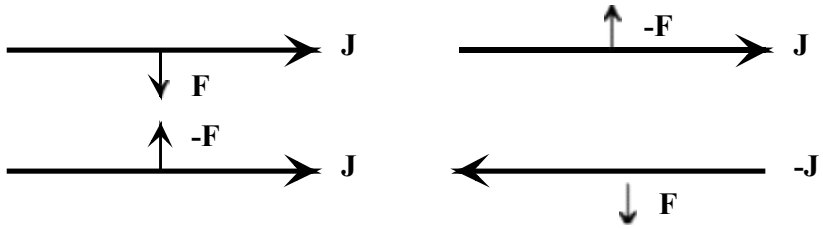


Figure 14.4 Direction of electric current and direction of force

A force exists between two wires, and this force is magnetic force. A similar phenomenon can be observed in a circular coil.

If two circular coils are placed parallel to each other with the same axis and current is passed through them in the same direction, it can be observed that the two coils attract each other, and if current is passed through them in opposite directions, they repel each other (Figure 14.5).

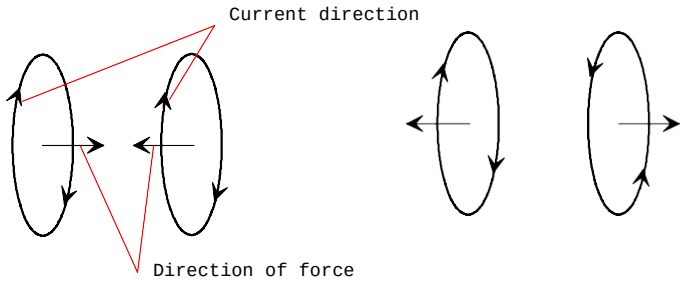


Figure 14.5 Forces between circular coils

A phenomenon very similar to this can be observed in experiments with fluids. What happens when you hang two pieces of paper parallel to each other and blow air between them? It seems reasonable to think that the two pieces of paper would move apart because the air would push them apart. However, experiments have shown that, as is well known, the two pieces of paper move closer together. Why is that? When you blow air, the airflow velocity between the two pieces of paper increases, but conversely, the air pressure decreases, causing the two pieces of paper to move closer together. It is well known that airplanes fly using this principle.

If two light ropes are moved quickly in parallel in the same direction, the surrounding air will be dragged along by the ropes due to the viscosity of the air. At this time, the air between the ropes is subjected to the force exerted by both ropes, and therefore experiences a greater force than the air outside the ropes. Consequently, the air between the ropes moves faster than the air outside the ropes. As the air between the ropes moves faster, the air pressure between the ropes becomes lower than the air pressure outside the ropes, and the ropes will be observed to pull on each other.

If two ropes are moved in opposite directions, the air-pushing forces of both ropes will cancel each other out. Conversely, the air pressure between the ropes will become relatively higher than the air pressure outside the ropes, and the ropes will appear to repel each other.

Similarly, if two light disks with the same axis are rotated in the same direction, it can be observed that they attract each other, and if they are rotated in opposite directions, they repel each other.

These phenomena are almost identical to those caused by magnetic forces resulting from electric current flowing through a wire. I have no hesitation in calling such phenomena fluid dynamics magnetic forces. It can be said that magnetic phenomena exist even within fluid dynamics.

In electrical magnetism, expressions like "two wires attracting each other" are used, but when considering hydrodynamic magnetism, it becomes clear that two ropes are not attracting each other, but rather are experiencing a force due to the pressure difference in the surrounding air. Electrical magnetism can also be thought of as being due to an electrical pressure difference occurring in space.

The idea of action at a distance often involves using non-essential expressions such as "mutually," which hinders the understanding of phenomena. For example, while hydrodynamic magnetism is simply the pressure difference of the surrounding air, it cannot be expressed so simply from an action-at-a-distance perspective. From a local action perspective, it can be stated that hydrodynamic magnetism is the pressure difference of the surrounding air.

When considering why a baseball curveball curves, it is almost impossible to explain it using action-at-a-distance mechanisms, but it is easy to explain it using action-near-field mechanisms. A baseball curveball curves because the airflow around the ball due to its rotation, combined with a relative wind flowing in the opposite direction of the ball's motion, creates an air pressure difference around the ball, causing it to curve (Figure 14.6; the figure ignores the complexity of the fluid).

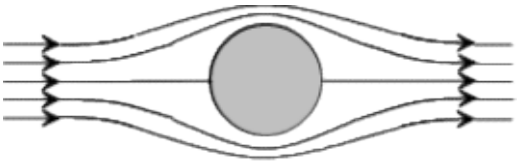


Figure 14.6(a) Flow around a ball flying without rotation

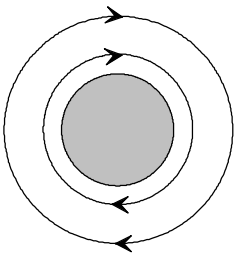


Figure 14.6(b) Flow around a rotating ball

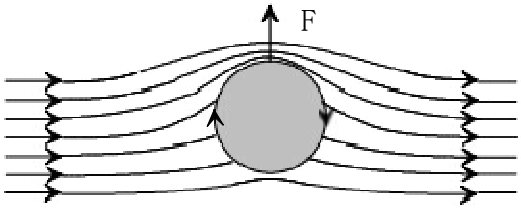


Figure 14.6(c) Flow around a ball in flight while rotating

Although the word "mutually" is never used in this context, it is clear that there is no essential difference between the force acting on the ball and the force acting between the two ropes. In other words, if a phenomenon like this baseball curveball is observed, it can be said that there is a magnetic effect at play.

14.8 Does Gravitational Magnetism Exist?

The vector $\mathbf{B}_g$ expressed by equation (14.35) is a mathematical representation of the field around a mass that can be considered simply as a single point moving at a constant velocity, and it does not say anything about the gravitational magnetic force between gravitational magnetic fields, which is of physical interest. In other words, while there is an effect of electrical magnetism in an electrical magnetic field, the existence of such an effect cannot be predicted from equation (14.35). Conversely, equation (14.35) holds true regardless of such an effect.

Electrical magnetism is something that has been confirmed through experiments, not through theory. Gravitational magnetism should also be confirmed through experiments. Nature provides evidence that this force exists.

Even with modern knowledge, the fundamental reason why gyroscopes precess is still unknown. Although this phenomenon has been observed for hundreds of years or more, physicists have yet to provide an answer. The gyroscope phenomenon has not traditionally been treated as part of the theory of gravity. However, because a gyroscope tries to maintain a constant orientation relative to the Earth, it is thought to interact with gravity, and can be considered a rotating gravitational field. From this perspective, the gyroscope phenomenon should be treated as part of the theory of gravity.

In the previous section's consideration of a fluid dynamics-based magnetic field, the true nature of that magnetic force was simply the pressure difference resulting from the relative velocity difference due to the synthesis of fluid flows in space. According to the principles of fluid dynamics, a force acts on a light sphere rotating in a constant-direction wind, where the effect of gravity is negligible. This is the same as a curveball in baseball.

Experiments have confirmed that such phenomena also exist in a gravitational field, if we consider Earth's gravity as a constant downward wind and a gyroscope as a rotating sphere (Figure 14.7).

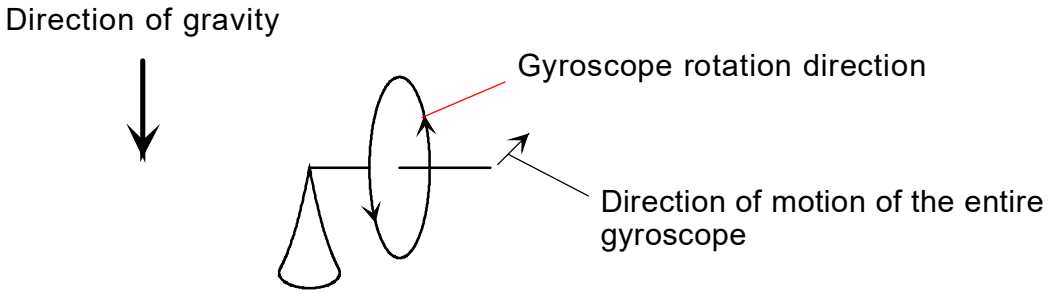


Figure 14.7(a) Gyroscopic motion

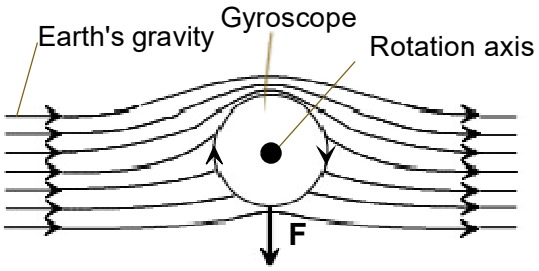


Figure 14.7(b) Forces acting on a gyroscope in the Earth's gravitational field

The precession of a gyroscope can be explained by considering it as some kind of pressure difference in the gravitational field resulting from the superposition of Earth's gravity and the rotating gravitational field caused by the gyroscope. However, while forces in fluid dynamics act relatively toward the direction of increasing velocity, experiments show that, contrary to magnetic forces in fluid dynamics, when the gyroscope's axis is placed parallel to the Earth's horizontal plane and rotated, the force in gravity acts from the direction where the downward Earth gravitational field and the gyroscope's rotation direction are superimposed and amplified toward the relatively weaker gravitational field.

In fact, the similarities between fluid dynamic magnetic phenomena and gravitational magnetic phenomena are noteworthy. If you rotate something as light as a ping-pong ball, where the effect of gravity is negligible, and change the direction of its axis in a constant wind, the ping-pong ball will experience forces in various directions. The direction of these forces is remarkably similar to the direction of forces a gyroscope experiences when its axis is changed on Earth, considering that the fundamental directions of the forces are opposite. For example, a bullet flies straight and stably when its axis of rotation is parallel to its direction of travel, just as a gyroscope rotates more stably when its axis of rotation is parallel to the direction of Earth's gravity. Such effects are one piece of evidence for the existence of gravitational magnetism.

Unlike hydrodynamic magnetism, gravitational magnetic force acts from the direction of amplified gravitational pressure towards the direction of relatively weaker pressure. Therefore, it is expected that two gyroscopes with the same axis of rotation will repel each other when rotated in the same direction, and attract each other when rotated in opposite directions. However, if this experiment is conducted on Earth, the gyroscopes will react more strongly to Earth's gravity, so a significant effect will not be observed. If such an experiment is to be conducted precisely, it should be done in a vacuum in a weightless environment.

14.9 Gravity-based propulsion systems

Is there a propulsion system that utilizes hydrodynamic magnetic force? Of course, a propeller cannot be used. A propeller does not use hydrodynamic magnetic force.

Such a device is similar to, for example, a rotating airplane wing; it simply involves rotating a cone-shaped object with the center of the lower circle as the axis from the apex. The reason why rotating such an object generates lift is due to the difference in surface area between the apex and the lower circle of the cone. In other words, the apex has a larger surface area than the lower circle, and therefore can exert more force on the surrounding air.

Therefore, the air pressure at the apex is lower than the air pressure at the lower circle, generating lift from the lower circle towards the apex. However, this device must be lightweight for the reasons described below.

What happens when this method is applied to gravitational magnetism? If an object with a conical mass distribution is rotated with the center of the lower circle as the axis from the apex of the cone, the direction of the force is opposite to that of hydrodynamic magnetism, so it is expected that gravitational magnetism will be generated from the apex of the cone toward the lower circle. When this force is actually generated in an experiment, it was confirmed to be present. The experiment involved attaching about five 5-yen coins(Translator's note:Japanese 5-yen coins have a hole in the center, like a washer.) with tape to one end of the axis of a commercially available gyroscope toy with a diameter of about 5 cm, and then bringing a polishing grinder into contact with the axis and rotating it at high speed. The gyroscope then moved from the area with a larger mass distribution toward the area with a smaller mass distribution. This experiment is simple enough for even elementary school students to do, so if you don't believe me, you should try it yourself.

The reason why it was said that the fluid dynamics device had to be made lighter was because the direction of the magnetic force in fluid dynamics and the magnetic force in gravity were opposite.

You might think, "So what if a gyroscope generates a gravitational magnetic force and moves, as in this experiment?" However, this means that humanity has acquired a way to control gravity. Gravity control is possible. I can say this with certainty, because it has been confirmed by experiment.

14.10 Ampère-Maxwell's Law in a Gravitational Field

Returning to Ampère's equation in a gravitational field (14.35), let us extend the equation to the form of the Ampère-Maxwell equation

$$c^2 \nabla \times \boldsymbol{B} = \frac{\boldsymbol{j}}{\varepsilon_0} + \frac{\partial \boldsymbol{E}}{\partial t}$$

, which is the non-stationary form of Ampère's law, and consider its analogous application to a gravitational field.

Maxwell obtained this equation by rewriting Ampère's law, but we can consider Ampère's law in a gravitational field in a similar way to Maxwell's method. This leads to its unstationary form, the equation in a gravitational field.

(1) Rewriting Ampère's Law into a non-stationary form using Maxwell's method (Electrodynamics)

Before Maxwell obtained the Ampère-Maxwell equation, the relationship between electric current and magnetic field was Ampère's law

$$c^2 \nabla \times \boldsymbol{B} = \frac{\boldsymbol{j}}{\varepsilon_0} \qquad \text{agein}(14.21)$$

That was all there was. Maxwell take div on both sides of equation (14.21),

$$\nabla \cdot (\nabla \times \mathbf{B}) = 0$$

Therefore,

$$\nabla \cdot \mathbf{j} = 0 \qquad (14.36)$$

He realized that this was strange. In fact, the equation,

$$\nabla \cdot \boldsymbol{j} = -\frac{\partial \rho}{\partial t} \qquad \text{agein}(14.10)$$

This is similar to the definition of the current $\mathbf{j}$, and since equation (14.21) is not a general form, he modified it as follows.

$$c^2 \nabla \times \boldsymbol{B} = \frac{\boldsymbol{j}}{\varepsilon_0} + \frac{\partial \boldsymbol{E}}{\partial t} \qquad (14.37)$$

If we take div in this expression,

$$\nabla \cdot \frac{\boldsymbol{j}}{\varepsilon_0} + \nabla \cdot \frac{\partial \boldsymbol{E}}{\partial t} = 0$$

$$\nabla \cdot \boldsymbol{j} + \varepsilon_0 \frac{\partial}{\partial t} \nabla \cdot \boldsymbol{E} = 0 \qquad (14.38)$$

Gauss's Law

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0}$$

Substituting this into equation (14.38)

$$\nabla \cdot \boldsymbol{j} = -\frac{\partial \rho}{\partial t}$$

This is consistent with equation (14.10). Therefore, equation (14.37) is considered correct, and no phenomena contradicting equation (14.37) have been found to date, so equation (14.37) is generally accepted.

(2) Application of gravity to mechanics

As already mentioned, Ampère's law in a gravitational field has already been derived in a similar form to that in electrodynamics. To reiterate,

$$c^2 \nabla \times \boldsymbol{B}_g = \frac{\boldsymbol{j}_g}{G_0} \qquad \text{again}(14.35)$$

Furthermore, from equation (14.19), the equation for continuity is:

$$\nabla \cdot \rho_g \boldsymbol{v} = -\frac{\partial \rho_g}{\partial t}$$

again(14.19)

Since we have obtained and can set $\mathbf{j}_g = \rho_g \mathbf{v}$, Equation (14.19) is in the same form as equation (14.38). It is natural that conclusions obtained in electrodynamics cannot be directly applied to the mechanics of gravity, but the mathematics of the field equations represented by equations (14.35) and (14.19) is exactly the same as that of electrodynamics, and in this case, mathematically, it is possible to perform the same operations as Maxwell's method. Therefore, if we rewrite equation (14.35) in the same way as Maxwell's method,

$$c^2 \nabla \times \boldsymbol{B}_g = \frac{\boldsymbol{j}_g}{G_0} + \frac{\partial \boldsymbol{E}_g}{\partial t} \qquad (14.39)$$

It can be written as. If you take div in this expression

$$\nabla \cdot \frac{\boldsymbol{j}_g}{G_0} + \nabla \cdot \frac{\partial \boldsymbol{E}_g}{\partial t} = 0$$

$$\nabla \cdot \boldsymbol{j}_g + G_0 \frac{\partial}{\partial t} \nabla \cdot \boldsymbol{E}_g = 0 \qquad (14.40)$$

Therefore, Gauss's law in a gravitational field is $\nabla \cdot \boldsymbol{E}_g = \frac{\rho_g}{G_0}$ Equation (14.40) is,

$$\nabla \cdot \boldsymbol{j}_g = -\frac{\partial \rho_g}{\partial t}$$

This is identical in form to equation (14.19). From this, it can be considered that the Ampère-Maxwell law, which is the transient form of Ampère's law in a gravitational field, is of the form of equation (14.39). This equation is in exactly the same form as the Ampère-Maxwell law in electrodynamics, but it is not uncommon for equations describing a field to be in exactly the same form. In this case, however, since the conclusion was obtained using the same mathematics as Maxwell's method, it is natural that the form is exactly the same. Let us call this equation (14.39) the Ampère-Maxwell equation in a gravitational field.

14.11 Magnetic Fields and Relativity

Magnetic fields are sometimes said to be an effect of the theory of relativity. It could be said that this view is mainstream among physicists. The basic idea is that if we consider an electric current as positive and negative charges moving in opposite directions at the same speed, then from the perspective of an observer stationary with respect to the wire, the relative velocity of the positive and negative charges is zero, and the electric field is zero. However, from the perspective of charges moving around the wire, the velocities of the positive and negative charges are not the same, and due to relativistic contraction, the densities of the positive and negative charges are different. As a result, from the perspective of the moving charges, an electric field is generated, and the moving charges experience a force. This force is said to be magnetism.

However, when we consider magnetic fields from a relativistic standpoint, a contradiction arises. The idea that moving an electrically charged object can be considered as generating an electric current and actually creating a magnetic field was proposed by Maxwell and confirmed by Roland's experiments.For example, if two charges $q_1$and q_2 , bearing the same sign and having the same magnitude, move in parallel at the same speed, they can be considered as two identical currents j_1 and j_2 flowing in parallel.Experiments have shown that currents flowing in the same direction attract each other; therefore, in this case, j_1 and j_2 will attract each other.However, from a relative perspective, the relative velocities of $q_1$and q_2 are zero, so from the perspective of q_1, q_2 is stationary, and from the perspective of q_2 , $q_1$is stationary.According to Coulomb's law, the force acting between stationary charges of the same sign is repulsive; therefore, $q_1$and q_2 will repel each other.Relatively speaking, to an observer stationary in the laboratory, j_1 and j_2 attract each other, so $q_1$and q_2 appear to move closer together. However, to an observer moving along with q_1 and q_2 , q_1 And q_2 will appear to be separated.Opponents may agree with this, but to us, it seems to us to be deeply contradictory.

According to our fluid dynamics thinking, such a contradiction does not arise. For example, according to Gauss's law, a positive charge can be considered as a source that continuously releases water radially at a constant rate. If two water sources w_1 and w_2 with the same release rate are placed side by side and kept at rest, the water flows between the two sources will cancel each other out. As a result, the velocity of the water between the sources will be slower, and the water pressure will be relatively higher than elsewhere. Therefore, a repulsive force acts between the two water sources.

Next, when water sources w_1 and w_2 are moved at the same velocity v, the vectors of the water flowing from w_1 and w_2 are combined with the velocity vector of the motion. Contrary to the case when the water sources are stationary, the amplification is strongest between water sources w_1 and w_2, the water flow becomes faster, and the pressure between the water sources becomes relatively lower than at other locations. Therefore, an attractive force acts on moving water sources (Figure 14.8).

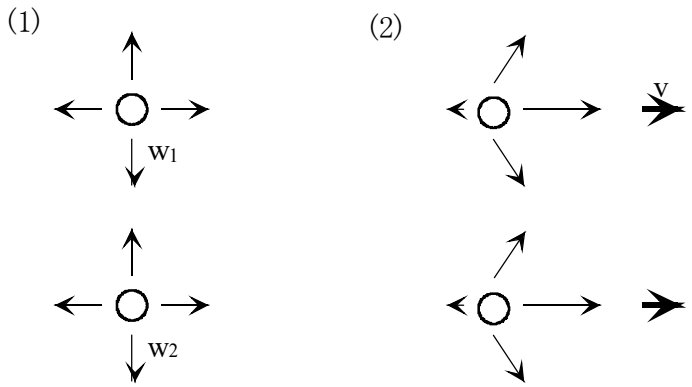


Figure 14.8 Synthesis of moving vectors

Applying this idea to electric charge eliminates the need for the absurd reasoning of relativists. If the moving electric field and the space through which it propagates are perfectly accompanied, they act as a repulsive force, as shown in Figure 14.7(1). If they are not perfectly accompanied, the electric field is left behind, and therefore no force acts on the electric charge. When magnetism exists, it can be considered an intermediate state between these two.