Chapter 12: The Law of Universal Gravitation

This chapter will explain the concept of conservative force fields related to Newton's law of universal gravitation. While the concepts presented here are not particularly new, they are fundamental to understanding the theory of gravity and should be recognized accordingly.

Towards the end of this chapter, the correct relationship between the law of conservation of kinetic energy and the law of conservation of momentum, which was a problem in Chapter 5, will be discussed. Furthermore, we will present an alternative to special relativity using the concepts of kinetic energy and potential energy.

12.1 Setting the Coordinate System

In certain locations where gravity exists, inertial acceleration is not zero, so those locations are not strictly inertial coordinate systems. If an object with mass exists, there will be a gravitational field in its vicinity, so the area around it is also not strictly an inertial coordinate system. Does a strictly inertial coordinate system even exist? For example, it has been mathematically proven that gravity is zero everywhere inside a spherical shell. If we adopt this idea, the inside of a spherical shell would be a strictly inertial coordinate system, or a wave-inertial coordinate system.

In an inertial coordinate system, if an object is moved with an initial velocity v_0, it will move in a straight line at a constant velocity within the coordinate system, and therefore, a scale can be placed on that coordinate system. The scale can be placed on the coordinate axes at equal time intervals, using the object's motion as a reference. To obtain equal time intervals, for example, the object can be rotated, and its rotation period can be used.

In non-inertial coordinate systems set in a gravitational field, for example, is it possible to easily mark the coordinate axes as in an inertial coordinate system? While a common time can be defined using the common time described in Chapter 1, measuring length is difficult in a gravitational field because, generally, objects do not move at a constant velocity in a straight line. Therefore, it is difficult to move an object at a constant velocity. Light is also thought to be bent in a gravitational field (bending means that its velocity changes), so measuring length using light as a reference cannot be easily affirmed. There is also the method of measuring length using a ruler made of a rigid body, but this length may change depending on the magnitude of gravity. This is because an object generally shrinks when a force is applied to it. Thus, the length of an object is thought to depend on the state of space, but in Chapter 3, we found that there are ways to measure length that do not depend on the state of space. This method uses triangulation. Using this method, it was possible to measure the relative difference in length without directly depending on the state of the space being measured. However, this method requires that the location being measured emits light or other light that indicates the object's position, and since this light can bend, strictly speaking, this method cannot accurately measure equally spaced distances. In conclusion, we must acknowledge that we currently do not know of a way to precisely measure equally spaced distances that are independent of the state of space. However, even if we don't know the method of measurement, there must be markings of equally spaced distances somewhere on an unmarked ruler placed in a gravitational field. For example, even if there are no clues whatsoever about the perpetrator of a bank robbery, we know that the perpetrator is either already dead or definitely exists somewhere in the universe. Therefore, even if we don't know a precise method of measurement, a coordinate system with such markings must exist. This idea is similar to Newton's absolute space, but these markings can be determined within the approximation range of triangulation and are not entirely without basis. Ultimately, all physical laws are approximations, and even an inertial coordinate system ceases to be strictly an inertial coordinate system if there is even one object with mass within that system.

We will call this approximate inertial coordinate system, which acknowledges the existence of gravity, the gravitational inertial coordinate system, and we will accept the existence of such a coordinate system. Such a coordinate system is fundamentally the same as the absolute spatial coordinate system that has been commonly used in Newtonian mechanics, but while existing physics has considered that time in a place with gravity may differ depending on the location, we will assume that this time is exactly equal by using a common time, and that although the length of an object in a gravitational field may change depending on the location, the measure of length in this coordinate system does not depend on the length or motion of the object.

12.2 The Law of Universal Gravitation

The law of universal gravitation, discovered by Newton, states that the magnitude of the gravitational force between two objects is proportional to their respective masses and inversely proportional to the square of the distance between them. Mathematically,

$$F = G\frac{mM}{r^2} \qquad (12.1)$$

This is the result. The G that appears in this is a type of proportionality constant called the gravitational constant. Its value is

$$\mathrm{G} = 6.6726 \times 10^{-11}\,[\mathrm{N \cdot m^2/kg^2}]$$

This value has been experimentally determined by Cavendish's experiment. Equation (12.1) can be written as a vector:

$$\boldsymbol{F} = G\frac{mM}{r^2}\boldsymbol{r_0} \qquad (12.2)$$

However, $\mathbf{r}_0$ is the unit vector in the r direction.

It is written as.

12.3 Representation of the Law of Universal Gravitation as a Field

Newton's law of universal gravitation defines the magnitude of the gravitational force between two objects with mass, but does the effect of this gravitational force propagate instantaneously between distant objects? According to the law of universal gravitation, the gravitational force is described as propagating instantaneously. If it takes some time for the effect of gravity to propagate, then the law of universal gravitation can be said to be a law that holds true in an approximation that considers the propagation time to be infinite. Alternatively, there are cases where such a law holds true even if the propagation time is not infinite. This is the case when the objects in question are not moving, and in this case, even if propagation time is taken, the propagating effect can be considered to remain unchanged, so there is no need to consider propagation time. Therefore, even in the approximation that the object that is the source of the gravitational force is not moving (a static gravitational field), the law of universal gravitation can be said to be correct.

The law of universal gravitation mathematically describes how two objects with mass instantaneously exert a force on each other across a vacuum. A vacuum is thought to be empty space, but the idea that forces can act even without any mediating substance is called action at a distance. In contrast, the idea that some force-transmitting effect propagates through a vacuum, and that this effect acts on an object, resulting in a force, is called action at close range. A vacuum may be empty space, but it clearly possesses the function of transmitting the effect of gravity.

The law of universal gravitation states that the closer you are to the source of gravity, the stronger the gravitational force is, and this state can be described as a field. Depending on the mass of an object, the way in which gravity exerts its force on its surroundings will differ. In other words, the characteristics of the gravitational field will differ. To express the gravitational field quantitatively, we can consider the force that would act on a unit mass brought to a point in the gravitational field without disturbing the original gravitational distribution.

For example, the force acting on an object P of mass m_P at any point h in a gravitational field, such as that of a planet that can be considered stationary, can be measured by Hooke's Law or Newton's second law of motion. If the force when this object is in free fall motion is F_P and the acceleration of object P is a, then that force is:

$$F_P = m_P a$$

Therefore, if the mass m_P is sufficiently small compared to the planet's mass, then the gravitational field created by object P can be considered to hardly disturb the planet's gravitational field. Next, suppose we drop another object Q of mass m_Q from the same place. If m_Q is not very large, it has been experimentally confirmed that the acceleration of object Q is the same as the acceleration a of P. That is, the force F_Q at this time is

$$F_Q = m_Q a$$

Therefore,

$$a = \frac{F_P}{m_P} = \frac{F_Q}{m_Q} \qquad (12.3)$$

This acceleration a is a quantity that depends on the location h within the gravitational field.

F_P and F_Q can also be measured directly using Hooke's Law or similar methods, without employing Newton's second law of motion. In this case, it is not necessary to subject the object to free fall motion, and acceleration 'a' is not used,

$$E_g \equiv \frac{F_P}{m_P} = \frac{F_Q}{m_Q} \qquad (12.4)$$

We can obtain the relationship. Let's call this E_g the strength of the gravitational field.

To precisely define the strength of a gravitational field, in order to avoid disturbing the original gravitational field, we can introduce an extremely small amount of mass and convert the force acting on it to a per-unit mass ratio. That is,

$$\boldsymbol{E}_g = \lim_{\Delta m \to 0} \frac{\boldsymbol{F}(\Delta m)}{\Delta m} \qquad (12.5)$$

It will be given as. If we convert this to per unit mass,

$$\boldsymbol{F} = m\boldsymbol{E}_g \qquad (12.6)$$

This means that $\mathbf{E}_g$ corresponds to equation (12.2),

$$\boldsymbol{Eg} = \frac{GM}{r^2}\boldsymbol{r}_0 \qquad (12.7)$$

Therefore, this quantity $\boldsymbol{E}_g$ is the same as the quantity of gravitational acceleration at r, as is clear from the correspondence between equations (12.3) and (12.4). That is,

[Strength of the gravitational field] = [Gravitational acceleration] (12.8)

That is the case.

12.4 Acceleration of a stationary object

In Chapters 8 and 10, we discussed the concept of acceleration from various perspectives, but here we will deepen our understanding of this concept further. In Chapter 8, we stated that we cannot consider the acceleration of a stationary object, but in reality, we can consider such acceleration. Equations (12.3) and (12.4) represent acceleration, but acceleration can also be defined as force/mass. If we consider acceleration as a function of time, we must conclude that when an object is not moving, its acceleration is zero. However, if we consider acceleration as force/mass, then acceleration can exist even when the object is not moving, as long as the applied force is not zero. Let's call this type of acceleration static acceleration. Using this static acceleration, we can consider the force applied to an object when it is not moving.

$$F = ma_0$$

a_0 is the static acceleration.

It can be set as.

12.5 Conservative Force Field

The work done by force $\mathbf{F}$ between A and B, W_{AB} , is given by equation (11.16):

$$W_{AB} = \int_A^B \mathbf{F} \cdot d\mathbf{s} \qquad \text{again}(11.16)$$

It can be written as. Substituting equation (12.6) into this equation,

$$W_{AB} = \int_A^B m\mathbf{E}_g \cdot d\mathbf{s}$$

Furthermore, if we set m = 1,

$$\varphi_{AB} = \int_{A}^{B} \mathbf{E}_g \cdot d\mathbf{s} \qquad (12.9)$$

This is the result. φ_{AB} is the work required to move a unit mass from A to B in the field $\mathbf{E}_g$.

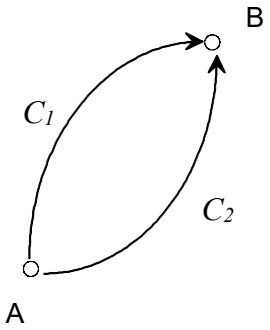


Figure 12.1 Path of line integral

Consider two different paths C_1 and C_2 connecting two points A and B. In the general case, the corresponding line integrals will have different values. However, when the values of these line integrals are the same for C_1 and C_2 (when the value of the line integral depends only on the starting point A and the ending point B), the vector field $\mathbf{E}_g$ is said to be conservative. When this vector is a force field, this field is called a conservative force field. The conditions for a vector $\mathbf{E}_g$ to be a conservative vector are:

$$\mathrm{rot}\mathbf{E}_g = 0$$

This applies only in that case and only at that time.

The difference between the line integrals of C_1 and C_2 is:

$$\varphi_{C1} - \varphi_{C2} = \int_{C1} \mathbf{E}_g \cdot d\mathbf{s} - \int_{C2} \mathbf{E}_g \cdot d\mathbf{s}$$

However, when $\varphi_{C1} - \varphi_{C2}$ is zero, $\mathbf{E}_g$ is a conservative force field. With respect to C_2 , the same path is taken, but the direction is changed from B to A, and the path C_2'

$$\int_{C_2'} \mathbf{E}_g \cdot d\mathbf{s} = -\int_{C_2} \mathbf{E}_g \cdot d\mathbf{s}$$

Therefore, If C is the closed loop A → B → A,

$$\varphi_{C1} - \varphi_{C2} = \oint_C \mathbf{E}_g \cdot d\mathbf{s} = \int_{C1} \mathbf{E}_g \cdot d\mathbf{s} + \int_{C_2'} \mathbf{E}_g \cdot d\mathbf{s}$$

Therefore,

$$\oint_C \mathbf{E}_g \cdot d\mathbf{s} = 0$$

This is the case of a conservative force field. That is, a conservative force field is a field subjected to a force of vector $\mathbf{E}_g$ where the line integral over one full turn is zero. Stokes' theorem, equation (11.40), is:

$$\oint_C \mathbf{E}_g \cdot d\mathbf{s} = \int_S \mathrm{rot}\mathbf{E}_g \cdot d\mathbf{S}$$

Therefore, in the case of a conservative force field,

$$\int_S \mathrm{rot}\mathbf{E}_g \cdot d\mathbf{S} = 0$$

And,

$$\mathrm{rot}\mathbf{E}_g = 0$$

It must be like that.

Conversely, if the rotation of vector $\mathbf{E}_g$ is not zero,

$$\mathrm{rot}\mathbf{E}_g \neq 0$$

A vector field $\mathbf{E}_g$ like the one shown is not a conservative vector. That is, when vector $\mathbf{E}_g$ is a rotational force vector, the field is not a conservative force field. Therefore, the field created by a rotating planet is not a conservative force field.

Since the Earth takes 24 hours to complete one orbit, its rotational speed is very slow, and it can be considered a static gravitational field where the Earth is not rotating. Furthermore, conducting an experiment on Earth means that the relative rotational speed between that location and the Earth is zero, so it can be considered that it is hardly affected by the Earth's rotation. The gravitational field created by the Earth as observed on Earth is subject to the equation $\mathrm{rot}\mathbf{E}_g = 0$, and such a field can be considered a conservative force field. This can be verified experimentally. For example, suppose we create a circular route within the Earth's gravitational field and move an object along this circuit. There are two ways to travel along this circuit, one being called rightward and the other leftward. If we move the same object with the same initial velocity and return to the starting point after one full rotation, and the speed is the same for both the rightward and leftward directions, then the circuit is a conservative force field. However, we are ignoring the resistance of the circuit. Even if we conduct this experiment at various locations within the Earth's gravitational field, tilting the circuit, no difference in speed occurs between the rightward and leftward directions. If there were a difference in speed, it would mean that energy could be extracted from the Earth's gravitational field.

A simpler experimental method involves placing a freely rotating disk with a uniform mass distribution at various locations within the Earth's gravitational field. If the disk begins to rotate, that location is not a conservative force field; if it does not rotate, that location is a conservative force field. Assume there is no rotational resistance.

12.6 Conservative Force Field of Wind

The concept of a conservative force field is applicable to general forces, and therefore, it is not limited to the gravitational field. For example, we can also consider the wind field. Since wind can exert a force on an object it strikes, the wind field is a force field, and if we write this force as $\mathbf{F}_w$, we can define work done.

$$W_{AB} = \int_A^B \mathbf{F}_w \cdot d\mathbf{s}$$

It can be written as. The condition for this wind field to be a conservative force field is,

$$\mathrm{rot}\mathbf{F}_w = 0$$

In this case, for example, a wind field blowing in a constant direction without vortices is a conservative force field. Even if a wind turbine like the one in Figure 12.2 is placed in this wind field, the wind turbine will not rotate.

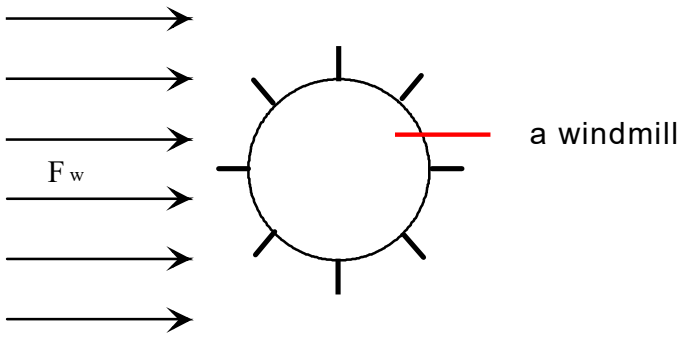


Figure 12.2 Conservative force field of wind

It is generally believed that energy cannot be extracted from conservative force fields such as gravity, but everyone knows how to extract energy from the conservative force field of wind. All you have to do is place a wind turbine, like a propeller, in the wind force field. The wind field around this propeller can be considered to be no longer a conservative force field locally because the direction of the wind force changes when the wind hits the propeller. Therefore, even in the wind field, energy is not being extracted from a conservative force field, but by installing a device like a propeller, the conservative force field is converted into a non-conservative force field. Converting a conservative force field into a non-conservative force field and extracting energy ultimately means that energy was extracted from a conservative force field.

Therefore, the idea that ancient scientists could extract energy from the gravitational field, a conservative force field, is not as absurd as modern people might think. If there were a clever mechanism in the gravitational field, like a wind turbine in the wind field, it should be possible to extract energy from it. Does such a mechanism exist? Ancient scientists all wanted to build such a device, and they worked on it seriously. However, as everyone knows, no one has actually built such a device yet.

When ordinary people work hard but are not rewarded, they want to give up their efforts. However, withdrawing without any reason would render all the efforts invested so far meaningless, so they have to come up with a reason to withdraw. What they do is make up a suitable reason. For example, they might fabricate the conclusion that such a device can never be physically realized. If this were true, the fact that the device could not be realized would not be their fault, because it had already been decided that it could not be realized before they even started the work. Or they might transform from a scientist into a philosopher by saying, "Something cannot be created from nothing." Or they might transform into a religious person by saying, "God would never allow such a thing." A serious person will realize that the reason it could not be realized is because they are incompetent and lose confidence. None of these methods are wise.

The current situation is that, having been deceived by the fabrications of past scientists, almost everyone believes that such a device cannot be realized. To put it bluntly, as stated in the introduction, we believe that it is possible to realize such a device. Readers are probably eager to know what kind of device it is. I don't mean to be coy, but I cannot describe it here because I am not yet fully prepared to explain this mechanism. I will discuss this in Chapter 15. What I wanted to say here is that the law of conservation of mechanical energy, which will be discussed later, does not state that energy cannot be extracted from a conservative force field.

12.7 The Concept of Potential Energy

Energy is needed to lift an object. Therefore, there is a belief that objects at higher altitudes have more energy than objects at lower altitudes. However, does this mean that objects at infinitely high altitudes (infinitely far from the source of gravity) have infinite energy? Not at all. The gravitational force at infinitely far distances (the point at infinity) is infinitely small, so it can be considered almost zero. Using this point at infinity as a reference, objects closer to the source of gravity require more energy to move to infinity than objects farther away. Conversely, objects closer to the source of gravity have more field energy added to them. Considering this, objects at lower altitudes have more energy than objects at higher altitudes.

When considering the quantity of field energy, it is convenient to use the point at infinity as a reference point, because the energy at this point can be considered zero. Such a convenient point does not exist near the source of gravity. For example, one could use the Earth's surface as a reference point and consider the potential energy at that point to be zero, but this method can describe phenomena relatively easily if the quantity of gravitational acceleration does not change with height. However, if it does change with height, it cannot be described in a simple way using the Earth's surface as a reference point.

In electrodynamics, there is a quantity called electric potential (see Chapter 13), and naturally, there is a quantity equivalent to this concept in the mechanics of gravity. Such a quantity is called the gravitational potential or simply the potential. If we write this quantity called potential as φ_g, it can be defined as follows:

$$\varphi_g = -\int_{\infty}^{r} \boldsymbol{E}_g \cdot \mathrm{d}\mathbf{s} \qquad (12.10)$$

The negative sign here indicates that if we consider the direction from the source of gravity towards infinity as the positive direction, then the direction of gravitational acceleration is the opposite, hence the negative sign. The way we define direction is merely for convenience, so expressing this equation without the negative sign is not necessarily incorrect.

The physical meaning of this equation is the work required to move an object of unit mass from position r to infinity in the field $\mathbf{E}_g$. In a conservative force field, the magnitude of φ_g is determined regardless of the path from r to ∞ (infinity), so φ_g can be considered a function of the position r in space. If the direction of $\mathbf{E}_g$ and the direction of motion of the object are the same,

$$\mathbf{E}_g \cdot \mathrm{d}\mathbf{s} = \mathrm{E}_g \mathrm{ds}$$
$$\mathrm{ds} = \mathrm{dr}$$
$$\mathrm{E}_g = \frac{GM}{r^2} \qquad (12.11)$$

Because of this relationship, when we calculate equation (12.10),

$$-\int_{\infty}^{r} \frac{GM}{r^2} dr = -\left[-\frac{\mathrm{GM}}{r}\right]_{\infty}^{r} = \frac{GM}{r} - \frac{GM}{\infty} = \frac{GM}{r}$$

Therefore, the potential $(\varphi_g)_r$ at a position r from the center of an object of mass M that can be considered spherical is:

$$(\varphi_g)_r = \frac{GM}{r} \qquad (12.12)$$

Therefore, the relationship between the strength of the gravitational field and the potential is:

$$\mathbf{E}_g = -\,\mathrm{grad}\varphi_g = -\nabla\varphi_g \qquad (12.13)$$

Therefore, since the spatial integral of $\mathbf{E}_g$ is φ_g, it is natural that the spatial derivative of φ_g is $\mathbf{E}_g$.

The quantity called positional energy U is defined as the product of the potential φ_g and the mass m of an object that can be considered a point, when the potential φ_g is brought to a location. That is,

$$U = m\varphi_g \qquad (12.14)$$

Often, potential energy is thought to be a quantity that describes the characteristics of a position in a gravitational field, but this is not entirely appropriate because potential energy changes depending on the mass of the object at that position. In contrast, potential has an intrinsic value for a spatial position in a gravitational field and is therefore suitable as a quantity that describes the characteristics of a position in a gravitational field.

12.8 The Physical Meaning of Potential

In Chapter 10, we discovered that the spatial integral of the concept of positional acceleration yields the concept of positional velocity. In Section 12.3, we used the law of universal gravitation to derive the concepts of gravitational strength and gravitational acceleration, focusing on the force aspect.

In this case, the same thing can be measured even if we use acceleration instead of force. The quantity $\mathrm{E_g}$, expressed in equation (12.11), is the acceleration due to gravity, and since this acceleration can be considered a function of space, it can be thought of as a type of position acceleration. That is, we can set **a** and $\mathrm{E_g}$ in equation (10.6) to be equal. Therefore,

$$\mathbf{a} = \mathrm{E_g} \tag{12.15}$$

Substituting equations (10.6) and (12.11), in this case ds = dr, and if we take the positive direction towards infinity,

$$\frac{d(\frac{1}{2}v^2)}{dr} = -\frac{GM}{r^2} \tag{12.16}$$

This differential equation is separable and can be solved simply by integration.

$$\frac{1}{2}v^2 = -GM\int\frac{1}{r^2}dr$$
$$\frac{1}{2}v^2 = \frac{GM}{r} + C$$

Therefore, if we give initial values for r = m and v = 0, then C = 0,

$$\frac{1}{2}v^2 = \frac{GM}{r} \qquad (12.17)$$

If we set $v_u = v$ and select the sign,

$$v_u = \sqrt{\frac{2GM}{r}} \qquad (12.18)$$

This is the result. This v_u is the velocity corresponding to the position in space. Let's call such a velocity the velocity of gravity or gravitational speed. The physical meaning of the velocity of gravity is the falling velocity of an object at a spatial position r in a gravitational field when it is freely falling toward the gravitational field under the initial conditions $r = \infty$, $v = 0$.

Also, from equations (12.12) and (12.17),

$$\varphi_g = \frac{1}{2}v^2 = \frac{1}{2}v_u^2 \qquad (12.19)$$

Thus, the potential can be converted as a function of velocity. Therefore, another physical meaning of the potential is the velocity of an object falling at a spatial position in a gravitational field, given that the initial condition for the object's velocity is zero at infinity.

12.9 Derivation of Gravitational Velocity

The gravitational velocity can be obtained without using the method described in 12.8. Assuming that the gravitational force at a position within a gravitational field is equivalent to the force of Newton's second law of motion, the gravitational force from a point P of mass M to a point Q of mass m at a distance r is $m\frac{GM}{r^2}$. If we consider the direction from P to Q as the positive direction, then...

$$m\frac{d^2r}{dt^2} = -m\frac{GM}{r^2} \qquad (12.20)$$

It can be written as. If we remove m from both sides,

$$\frac{d^2r}{dt^2} = -\frac{GM}{r^2} \qquad (12.21)$$

This is the result. Although this relationship has been implicitly accepted, it directly links the force according to Newton's second law of motion with the force according to the law of universal gravitation. How can these two separate forces be considered equal? This can be seen, for example, by conducting experiments like those in equations (12.3) and (12.4). Both sides of equation (12.21) represent acceleration, which can also be defined as force/mass. When expressed in this way, the acceleration of Newton's second law of motion and the acceleration of the law of universal gravitation can be expressed in exactly the same form (force/mass).

set $v=\frac{dr}{dt}$ then $\frac{d^2r}{dt^2}=v\frac{dv}{dr}$, equation (12.21) is

$$v\frac{dv}{dr}=-\frac{GM}{r^2}$$

$$vdv=-\frac{GM}{r^2}dr \quad \text{(Separable variables)}$$

$$\frac{1}{2}v^2=-\frac{GM}{r}+C$$

If we give the initial condition v = 0 when r = m, that is, at the point at infinity...If it is accelerated only by gravity, then C = 0,

$$\frac{1}{2}v^2=-\frac{GM}{r} \qquad (12.22)$$

This result is the same as equation (12.17),

$$\varphi_g=\frac{1}{2}v^2$$

This is the result.

12.10 Law of Conservation of Mechanical Energy

In equation (12.22), if we give the initial value v = v_0 when r = ∞, then $C = \frac{1}{2}v_0^2$, equation (12.22) is,

$$\frac{1}{2}v^2 - \frac{GM}{r} = \frac{1}{2}v_0^2 \qquad (12.23)$$

It can also be written as. Multiplying both sides of this equation by the mass m gives what is known as the law of conservation of mechanical energy. That is,

$$\frac{1}{2}mv^2 - m\frac{GM}{r} = \frac{1}{2}mv_0^2 \qquad (12.24)$$

The first term on the left side of this equation is called kinetic energy, and the second term is called potential energy. The right side is constant and does not change, so we write kinetic energy as T, potential energy as U, and energy as E.

$$T + U = E = \text{constant} \qquad (12.25)$$

It is sometimes written as such.

A moving object possesses energy. This is a quantity usually called momentum, and is written as $P = mv$. Why is it necessary to write a similar quantity in a more complex form such as $T = \frac{1}{2}mv^2$?

Originally, equation (12.24) was derived from equations (12.16) and (12.20). Equations (12.16) and (12.20) are so-called differential equations, while equation (12.24) is an algebraic equation. Since algebraic equations are simpler than differential equations, the energy of a moving object is simply written as $\frac{1}{2}mv^2$ instead of using mv. Of course, it is also possible to write something similar to equation (12.24) using mv. From equation (12.20),

$$\frac{d(mv)}{dt} = -m\frac{GM}{r^2} \qquad (12.26)$$

That's all you need to write.

Differentiating $\frac{1}{2}\mathrm{mv}^2$ with respect to v gives mv, so the kinetic energy T and momentum P are..

$$\mathrm{P}=\frac{dT}{dv} \qquad (12.27)$$
$$\mathrm{T}=\int \mathrm{Pdv}$$

The relationship is as. From the relationship in equation (10.8),

$$\frac{d(\frac{1}{2}mv^2)}{ds}=\frac{d(mv)}{dt}$$

Therefore,

$$\frac{\mathrm{dT}}{\mathrm{ds}}=\frac{\mathrm{dP}}{\mathrm{dt}} \qquad (12.28)$$

It can also be written as. Differentiating momentum with respect to time yields force, but differentiating kinetic energy with respect to space also yields force. A quantity differentiated with respect to time is a function of time, but a quantity differentiated with respect to space is a function of space. Since potential energy is a function of space, it is easier to compare it using kinetic energy as a function of space than using momentum as a function of time. If we express equation (12.24) in terms of force,

$$\frac{d(\frac{1}{2}mv^2)}{dr}-\frac{d(m\frac{GM}{r})}{dr}=\frac{d(\frac{1}{2}mv_0^2)}{dr} \qquad (12.29)$$

This is the result. Each term in this equation represents a force, and this equation is a force-based expression of the law of conservation of mechanical energy, and is equivalent to equations (12.16), (12.20), and (12.24).

Equation (12.24) can be rewritten from equation (12.19) as follows:

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_u^2 = \frac{1}{2}mv_0^2 \qquad (12.30)$$

The first term v on the left side is the velocity of the object when it is in free fall, and the second term v_u is the velocity defined as a function of space using the velocity of the object when it is in free fall. Therefore, if we give an initial value of $v_0 = 0$, it is natural that $v=v_u$. Equation (12.30) does not provide a relationship between kinetic and potential energy in any given motion, but is only applicable in the case of free fall.For example, if an object is at rest in a suitable location within a gravitational field, $v = 0$. Substituting this into equation (12.30) gives $v_u^2 = -v_0^2$. This is, of course, incorrect.

If we consider that total energy is the sum of the absolute values of kinetic energy and potential energy, then

$$\frac{1}{2}mv^2 + \frac{1}{2}mv_u^2 = \frac{1}{2}mv_e^2 = E \qquad (12.31)$$

v_e is the velocity with the total energy converted, and E is the total energy

.It can be expressed as. For example, if an object is at rest in a suitable location within a gravitational field, then $v = 0$, so $E = \frac{1}{2}mv_u^2$, meaning that the total energy of that object is only potential energy. Equation (12.31) can be thought of as the total energy of an object undergoing arbitrary motion within a gravitational field.

12.11 What is the Law of Conservation of Kinetic Energy?

In Chapter 5, when discussing the relationship between the law of conservation of momentum and the law of conservation of kinetic energy, I pointed out that the existing understanding in physics is incorrect. That error is:

$$\mathbf{P} = |m_1 \mathbf{v}_1| + |m_2 \mathbf{v}_2| + ... + |m_n \mathbf{v}_n| \quad (12.32)$$

$$T = \tfrac{1}{2} m_1 \mathbf{v}_1^2 + \tfrac{1}{2} m_2 \mathbf{v}_2^2 + ... + \tfrac{1}{2} m_n \mathbf{v}_n^2 \quad (12.33)$$

The idea was that both formulas could coexist.

What could be wrong? Equation (10.8) is,

$$\frac{d(\frac{1}{2}mv^2)}{ds} = \frac{d(mv)}{dt}$$

This was expressed as, but if we rewrite this equation in integral form,

$$\frac{1}{2}mv^2 = \int mv\frac{ds}{dt} = \int mvdv \quad (12.34)$$

This is one example of the law of conservation of momentum (12.32).

$$m_1 v_1 = m_2 v_2 + m_3 v_3 \quad (12.35)$$

It can be written as, but if we integrate this equation with respect to v,

$$\int m_1 v_1 dv = \int m_2 v_2 dv + \int m_3 v_3 dv \quad (12.36)$$

This is the result. This equation cannot be integrated with respect to v, but v_1, v_2, and v_3 are functions of v, and they are

$$v_1 = av \tag{12.37}$$

$$v_2 = bv$$

$$v_3 = cv$$

However, a, b, c is the constant of proportionality

We can set this as. Substituting this into equation (12.36),

$$\int m_1 avdv = \int m_2 bvdv + \int m_3 cvdv$$

Therefore, we can integrate with respect to v. When we integrate,

$$\frac{1}{2}m_1 av^2 = \frac{1}{2}m_2 bv^2 + \frac{1}{2}m_3 cv^2 \qquad (12.38)$$

If the law of conservation of kinetic energy is correct,

$$\frac{1}{2}m_1 (av)^2 = \frac{1}{2}m_2 (bv)^2 + \frac{1}{2}m_3 (cv)^2 \qquad (12.39)$$

It should be (12.39), but equations (12.38) and (12.39) are not equal, and equations (12.32) and (12.33) are incompatible. What exactly is the basis for the law of conservation of kinetic energy? Probably, a result like equation (12.30) was roughly extended to represent the relationship between individually moving objects. Equation (12.30) is valid, but this equation describes the relationship between the velocity of a moving object and the velocity of gravity, not the relationship between moving objects themselves, and from this equation, a result like equation (12.33) cannot be obtained. The law of conservation of kinetic energy is wrong, such a thing is not a law, and the correct form is equation (12.38).

Substituting equation (12.38) back into the term of equation (12.37),

$$\frac{1}{2}m_1 v_1 v = \frac{1}{2}m_2 v_2 v + \frac{1}{2}m_3 v_3 v \qquad (12.40)$$

Therefore,

$$m_1 v_1 = m_2 v_2 + m_3 v_3$$

Therefore, since this equation is the law of conservation of momentum itself, it is almost meaningless to write it in the form of equation (12.38).

12.12 Representation of Gravitational Velocity by the Speed of Light

What follows is not necessary for the theory of gravity that will be discussed later. Therefore, it is not of much importance to us. In Chapter 4, we completely rejected special relativity, but we did not offer an alternative. The rejection of special relativity is necessary for our theory of gravity, but an alternative is not. However, this attitude is a little dishonest, so let us at least state what we consider to be an alternative. We thought that the total energy in the gravitational field of an object of any mass in motion would be expressed by equation (12.31). Assuming that the motion of light can also be applied to this equation,

$$\frac{1}{2}mc_r^2 + \frac{1}{2}mv_u^2 = \frac{1}{2}mv_e^2 \qquad (12.41)$$

However, c_r is the speed of light corresponding to the position in the gravitational field.

Since $\mathrm{v_u} = 0$ at far infinity,

$$\frac{1}{2}mc_r^2 = \frac{1}{2}mv_e^2 \qquad (12.42)$$

Therefore, in this case, we can assume that light propagates at a constant speed c.

$$v_e = c \qquad (12.43)$$

Therefore, if we substitute this equation into equation (12.41),

$$\frac{1}{2}mc_r^2 + \frac{1}{2}mv_u^2 = \frac{1}{2}mc^2 \qquad (12.44)$$

That is,

$$c_r^2 = c^2 - v_u^2 \qquad (12.45)$$

So, if we rewrite this equation...

$$c_r = c\sqrt{1 - \frac{v_u^2}{c^2}} \qquad (12.46)$$

This can be written as. According to this equation, the greater the gravitational potential (the closer to the source of the gravitational field), the slower light travels. If we define the scale of time and length using the speed of light, then since light travels more slowly closer to the source of the gravitational field, we can consider that time flows more slowly and the scale of space length also contracts.

Furthermore, in order to bring an object that is free-falling from infinity with zero initial velocity to rest in a suitable location within a gravitational field, it must be accelerated in the opposite direction with energy commensurate with the gravitational velocity at that location. If an accelerometer were installed on the object at this time, the value immediately after it comes to rest would indicate an inertial velocity equal to its gravitational velocity. In other words, an object moving at a constant inertial velocity can be considered to be in the same state as being at rest in a location with a gravitational velocity commensurate with its inertial velocity. Considering this, it can be thought that for an object moving with inertial velocity, the flow of time slows down and the scale of spatial length is compressed, and this relationship can be considered as v_u in equation (12.46) being taken as the inertial velocity.

In equations (12.44) and (12.46), when $v_u > c$, c_r becomes negative. In a gravitational field where the mass is large and concentrated at the center, there may actually be places where $v_u > c$. For example, the position around the Earth where $v_u = c$ is given by equation (12.18) $c = \sqrt{\frac{2GM}{r}}$,

$$r = \frac{2GM}{c^2} = \frac{2 \times 6.67 \times 10^{-11} \times 5.977 \times 10^{24}}{(3.0 \times 10^8)^2} = 8.86 \times 10^{-3}[m]$$

Therefore, if we imagine the Earth as tiny as the tip of a needle while maintaining its mass, then the area about 8 mm away from it would be $v_u > c$.

Would the time and space of such places also become negative? We don't know the answer to that. Furthermore, it's not very meaningful to even consider it. Since humanity can't even go to Mars, there's no way to verify such things.