Chapter 8: Force and Energy

~Accelerated force and force with zero acceleration~

When opposing forces exist, equilibrium is maintained, and no work is done. Modern physics posits that in this situation, there is no change (consumption) of energy. Many phenomena demonstrate this, including tug-of-war where two forces pull with equal force, supporting a weight with your hands, and a weight placed on a table.

When you hold and support a weight with your hand, your arm muscles become fatigued, and it seems as though you need more energy than when you're doing nothing. According to physicists, this perceived energy requirement is due to the biological structure of the muscles, and some types of muscles do not consume energy to support weight.

However, is it really acceptable to leave it at such a simple explanation? First, let's examine the phenomena related to force and clarify the problems.

8.1 Consideration of phenomena related to force

When you attach a string or something similar to an object placed on a table and pull it, there is generally friction (resistance) between the object and the table, and you cannot move the object without applying a certain amount of force. This friction can take various forms, from small ones like air resistance to those requiring large forces like the breaking strength of steel frames.

When resistance is low, a small force is needed to move an object, but when resistance is high, a large force is required to move it. An observer observes a dramatic change between the moment an object begins to move and the moment before. According to physicists, this is because the work done has changed from zero to something other than zero.

Modern physics concludes that the work done before an object starts moving is zero. Since work is another way of expressing energy, it must be concluded that the added energy is also zero. If the added energy is zero, then it is the same as when no force is applied. Clearly, the physical state of an object is not the same when it is being pushed and when it is not being pushed. However, the mathematical representation of these states is the same. This suggests that the phenomenon cannot be mathematically represented correctly.

In fact, it should be obvious that an object subjected to force is not the same as an object that is not subjected to force. If force is applied to an iron rod, the rod will begin to bend after a certain amount of force, but this change does not happen suddenly. With a small force, the rod will deform and generate heat to an extent that is barely observable. If the rod deforms and generates heat, everyone would say that energy was required. Yet, even in modern physics, it is argued that no energy is required until the rod begins to bend.

It is not uncommon in physics for the effects applied and the observed phenomena not to be in a simple proportional relationship. Heating ice at -20° C by 10° C shows almost no change, but heating it another 10° C results in a dramatic change. This phenomenon is similar to the friction phenomenon between objects. Although these phenomena seem completely different, the melting of ice and the bending of a rod can be considered to have essentially the same cause if we consider that the rod begins to bend because it has partially melted. In both cases, what was done before the dramatic change occurred may seem to have done no work, but it turns out that it was actually necessary.

8.2 What is Force?

What is force? Anyone familiar with Newton's second law of motion would answer that force is the product of mass and acceleration. That is,

$$\mathbf{F} = m\mathbf{a} \tag{8.1}$$

On the other hand, someone with more knowledge might answer that force is not defined solely as mass × acceleration, but also as being defined by laws such as the law of universal gravitation and Coulomb's law. In other words, it's not strictly correct to say that force is the product of mass and acceleration; more accurately, force can be expressed as the product of mass and acceleration. The point is that the concept of force and the product of mass and acceleration are not directly related; it's simply that this expression allows for quantitative measurement.

Newton's second law of motion can also be expressed as follows:

$$\mathbf{F} = \frac{d\mathbf{P}}{dt} \tag{8.2}$$

Which **P** is momentum, and It is **P**=m**v**.

The right-hand side of this equation represents the change in momentum over time, so one characteristic of force is that it is an effect that changes momentum over time.

8.3 Force and Acceleration

It is said that everything in classical mechanics can be explained by Newton's laws of motion. These laws have long been supported and are considered strictly true except in the realm of relativity. However, it is thought that these laws cannot describe all phenomena of classical mechanics.

The force defined by $\mathbf{F}=m\mathbf{a}$ is equal to mass $\times$ acceleration. However, when the object in question is at rest, its acceleration is clearly zero, and therefore, it must be concluded that no force exists.

For example, if a weight is stationary on a table on Earth, gravity acts on the weight, pushing it against the table, and the table, in reaction, pushes back against the weight. In this case, the force pushing the table and the reaction force are forces of classical mechanics and must be considered to follow Newton's laws of motion. If this force pushing the table follows Newton's second law of motion, then that force can be written as $\mathbf{F}=m\mathbf{a}$.

Suppose we want to know the force with which a mass pushes against a table, according to Newton's second law of motion. According to Newton's second law of motion, where there is a force, there must be an acceleration. If we measure the position of the mass as a function of time using an appropriate coordinate system, we can observe that the mass is "stationary," and therefore there is no change in the position of the mass with respect to time. Consequently, the acceleration of the mass is zero. Here, we cannot introduce an interpretation such as the acceleration due to the force of the mass being small and somehow canceling out the acceleration due to the reaction force from the table.

Our thought process involves deriving conclusions from Newton's second law of motion, but such interpretations clearly cannot be derived from Newton's second law of motion.

As a result, since the acceleration of the mass is zero, the conclusion that no force is acting on the mass can be obtained using Newton's second law of motion. This clearly contradicts observational facts. Therefore, it must be concluded that Newton's laws of motion cannot describe all the forces of classical mechanics.

A similar example is when two spheres are in contact and at rest. It's impossible to distinguish whether they are merely lightly touching or strongly pushing against each other due to some effect by measuring their acceleration. According to Newton's second law of motion, since there is no acceleration, we must conclude that there is no force.

8.4 Force with Zero Acceleration

According to Newton's second law of motion, when acceleration is zero, force is also zero. If this were true, there would be no force between the table and the weight. However, observational data indicates that there is a force there. What law allows us to say this?

That law is called Hooke's Law.

$$F = kx \qquad (8.3)$$

k is the proportionality constant of the displacement (e.g., the spring constant), and x is the displacement.

This law is written as. This law is an experimental law that measures the magnitude of force depending on the properties of the material (for example, something like a spring), but it is very useful in practice, and in fact, Newton's second law of motion is rarely used when measuring force, and even forces that cannot be measured by Newton's second law of motion, such as forces with zero acceleration, can be measured according to this law.

In this way, we measure a force with zero acceleration. It is impossible to divide the force obtained in this manner into mass × acceleration, and from this, we can see that force is not mass × acceleration, but rather a quantity in itself.

By the way, can we consider a force with zero acceleration and a force with non-zero acceleration to be the same force? What basis can we have for considering these two forces, which exhibit distinct characteristics, as the same force?

The problem lies in how to reconcile a force with zero acceleration with a force that does not have zero acceleration. Because this has been done with an ambiguous understanding, physicists have been unable to comprehend the concept of force.

8.5 Force and Energy

How do physicists understand the relationship between force and energy?

Returning to Newton's second law of motion, force can be interpreted as the effect of changing momentum. If momentum changes, then there is a change in energy. Therefore, it can be stated that force is the effect of changing energy.

For now, let's assume that force is the effect of changing energy, as this can be said from Newton's second law of motion. Newton's second law of motion cannot be applied to forces with zero acceleration, so it can be argued that this interpretation cannot be applied to forces with zero acceleration. In fact, the modern physics' understanding that forces with zero acceleration do not require energy seems to be a conclusion derived from Newton's second law of motion.

If that were the case (if a force with zero acceleration requires no energy), then a force with acceleration and a force with zero acceleration must be considered different types of forces that cannot be quantitatively compared or related to each other. This is because a force with acceleration changes energy, while a force with zero acceleration does not, exhibiting different characteristics.

Let me explain in more detail. Suppose an object, which can be considered a point mass of mass m, is moving at a constant velocity along the x-axis with an initial velocity v_0 in a space where there are no external forces. A force F acts on this object in the direction of the x-axis at time t = 0, and thereafter, the magnitude and direction of this force remain constant. The object is accelerated by the action of the external force. The motion at time t = 0 is, according to Newton's second law of motion,

$$F = m\frac{d^2x}{dt^2} \qquad (8.4)$$

It is expressed as. The integral of acceleration $\frac{d^2x}{dt^2}$ with respect to

time t is the velocity.

$$\int_0^t \frac{d^2x}{dt^2}\,\mathrm{dt} = \int_0^t \frac{dx}{dt} = \int_0^t dv \qquad (8.5)$$

Therefore, if we let the velocity at time t = 0 be v_0 and the velocity at time t be v,

$$\int_0^t \mathrm{dv} = \int_{v_0}^{v} \mathrm{dv} = v - v_0 \qquad (8.6)$$

Therefore, Substituting equation (8.4) into equation (8.5),

$$\int_0^t \frac{d^2x}{dt^2}\,dt = \int_0^t \frac{F}{m}\,dt = \frac{F}{m}t \qquad (8.7)$$

Therefore, from equations (8.5), (8.6), and (8.7),

$$\mathrm{Ft} = \mathrm{mv} - \mathrm{mv_0} \qquad (8.8)$$

This equation represents the difference in momentum (force × time), and the right-hand side, in other words, represents the change in energy. Therefore, the left-hand side of equation (8.8) is a representation of energy in the form of the product of force and time. Ft can be thought of as the integral of F with respect to t, and such a quantity is called impulse.

$$\text{impulse} = \int_0^t F dt \qquad (8.9)$$

It is written as.

It is often said that impulse is defined only when the force is very large and the duration of its action is very short, but this interpretation is not essential. Not only does it fail to answer questions such as how large the force is and how short the duration of its action is, but by definition, it can also be true when the force is small and the duration of its action is long. It is likely that such a sloppy interpretation by physicists stems from the misconception that "a force with zero acceleration should not require energy." In this case, since the concept of impulse is derived from Newton's second law of motion, it should be said that it should only be defined for forces with acceleration.

However, the quantity impulse is not inherently derived from Newton's second law of motion. Generally, if we graph a quantity F and time t, it will look like Figure 8.1, and the area E enclosed between this graph and the t-axis is,

$$E = \int_{t1}^{t2} F dt = [Ft]_{t1}^{t2} \qquad (8.10)$$

It can be expressed as. In this case, F is a quantity that can be measured using Hooke's Law, etc., regardless of whether or not the object is accelerated by this force, and is not directly related to Newton's second law of motion.

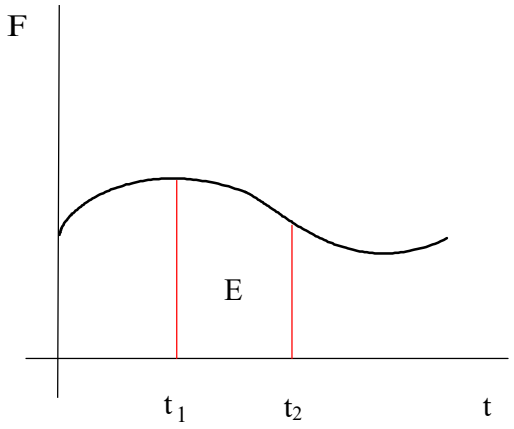


Figure 8.1 Graph of Force and Time

The F that appears in equation (8.8) is a quantity called force, but according to the interpretation of modern physicists, this time integral of force is valid only for forces with acceleration, and not for forces with zero acceleration. That is, if we let the energy be E,

$$E = \int_0^t F_a dt \qquad F_a \text{ is accelerated force} \qquad (8.11)$$

$$E \neq \int_0^t F_0 dt = 0 \qquad F_0 \text{ is force with zero acceleration} \qquad (8.12)$$

That's how it works.

In response to this, physicists might argue that "force in modern physics is defined as something that should be expressed by equations (8.11) and (8.12), and the concepts of energy and work used in everyday life are different from those used in physics." However, as is clear from the comparison with equation (8.10), this quantity is not zero. Therefore, we cannot accept that equation (8.12) is valid. If we accept that F_a and F_0 are the same force, then a revision of modern physics is required.

8.6 Relationship between Forces with Zero Acceleration and Forces with Non-Zero Acceleration

How can we connect a force with zero acceleration with a force that does not have zero acceleration? This can be investigated relatively easily through experimentation using Newton's second and third laws of motion and Hooke's law. For example, when pushing an object, a spring scale can be used to determine the magnitude of the force. Even when the object does not move, the spring scale reading indicates that a force is being applied, and its magnitude can be measured. Using this quantity obtained with the spring scale, we can compare the force when acceleration is zero with the force when acceleration is not zero. If the quantities are the same, we can conclude that the same force was applied. While this understanding is implicit, the relationship between a force with zero acceleration and a force with acceleration is not directly derived from Newton's laws of motion; Hooke's law is necessary, and it is by no means self-evident. This is a law that connects Newton's laws of motion and Hooke's law through experimentation. That is,

$$F_a \propto F_0 \tag{8.13}$$

$$F_a = F_0 \quad \text{the units of force between the two parties should be chosen} \tag{8.14}$$

It is expressed as.

onsidering only the force aspect, equation (8.13) would likely hold true experimentally. However, considering the energy aspect, while it is clear that equation (8.12) is not zero, it has not yet been clarified whether this quantity can be considered energy. As physicists suggest, a force with zero acceleration may not require energy. Let's consider this point next.

8.7 Is energy required for a force applied to an object when it is not moving?

Does applying a force with zero acceleration require no energy? To answer this question, let's pose the following problem.

[Problem] Is the energy required to keep an object of mass m at rest for t seconds in a place with gravitational acceleration g equal to the energy required to continuously accelerate an object of mass m with acceleration g for t seconds in a place without gravity?

From the perspective of modern physics, the answer to this problem is that they are not equal, because energy is not required to keep them at rest.

Now, let's prove that the answers to this problem are equal.

[Basic Prerequisites]

Let us accept the idea of the equivalence principle. This idea states that if an elevator floating in zero gravity is continuously pulled with an acceleration of g, which is the acceleration due to gravity on Earth, then the environment inside the elevator can be considered equivalent to an environment like the gravitational field on Earth.

To digress slightly, let me explain the equivalence principle here. The equivalence principle is thought to play an important role in Einstein's theory of relativity, but this idea is fundamentally contradictory to the concept of relative force. The equivalence principle states that when you pull an elevator, it appears as if there

is a gravitational field inside the elevator, but what is the reference point for the acceleration used to pull this elevator? For example, suppose observer P is inside a stationary elevator on Earth, and observer Q is outside the elevator, and Q is running sideways with acceleration b. From Q's perspective, it appears that a relative acceleration of b is acting on P. In this case, will P be pushed sideways with acceleration b? Experiments show that no force acts on P (ignoring Earth's gravity). However, from Q's perspective, a relative acceleration of b is acting on P. If all motion is a relative concept, then how can we distinguish this relative acceleration from the acceleration inside the elevator when it is actually being pulled? There is absolutely no way to do this.

Readers have probably already noticed. What we've discussed here is the same concept of inertial acceleration that we discovered in Chapter 2. The equivalence principle doesn't affirm relativity, but rather absoluteness. Einstein discovered a principle that contradicted his own theory and incorporated it into his theory. He dug his own grave.

Since the equivalence principle and the concept of inertial acceleration are the same thing, the acceleration pulling the elevator must be inertial acceleration.

[Proof]

Assume there is an elevator of mass M in a zero-gravity environment. Let S' be a fixed coordinate system relative to this elevator, and assume the elevator is moving in a straight line with constant acceleration g relative to another coordinate system S. (More accurately, this acceleration must be inertial acceleration.) There is an object of mass m inside the elevator, and this object has zero relative velocity to S', that is, it is at rest relative to the elevator. In this case, from the perspective of S', this object is equivalent to being at rest on Earth (by the equivalence principle).

The equation of motion as seen from S is:

$$F_1 = (M+m)g \tag{8.15}$$

Therefore, the energy required for t seconds is, from equation (8.11),

$$E_1 = \int_0^t F_1 dt = \int_0^t (M+m)gdt \tag{8.16}$$

That is the case.

Next, remove the objects inside the elevator and repeat the same process. At this point, the equation of motion as seen from S is:Therefore

$$F_2 = Mg \tag{8.17}$$

Energy is,

$$E_2 = \int_0^t F_2 dt = \int_0^t Mgdt \tag{8.18}$$

Therefore, perspective from S', in the phenomenon of equation (8.16), the object was kept at rest, whereas in the phenomenon of equation (8.18), the object was not kept at rest. Thus, equation (8.16) - equation (8.18) is the energy required to keep the object at rest. That is, this energy is

$$E_1 - E_2 = \int_0^t mgdt \tag{8.19}$$

This is the result. On the other hand, in the absence of gravity, the energy required to continuously accelerate an object of mass m with an inertial acceleration of g for t seconds is given by equation (8.11):

$$E_3 = \int_0^t mgdt \tag{8.20}$$

Therefore, Equations (8.19) and (8.20) are equal. Thus,

"The energy required to keep an object of mass m at rest for t seconds in a place with gravitational acceleration g is equal to the energy required to keep an object of mass m in a place without gravity to continuously accelerate with an acceleration of g (more accurately, inertial acceleration) for t seconds”

There is no doubt that "applying force to an object causes some kind of change in the object." It can also be acknowledged that "energy is necessary to cause a change in an object." In other words, it is almost impossible to avoid concluding that "energy is necessary to apply force." If there is a force between an object placed on a table and the table, then it can be assumed that energy is necessary.

8.8 Analogy of Wind

Our understanding is that applying a force to an object requires energy, regardless of whether the object is accelerating or not. Let's consider this further using wind, which is the flow of air.

Suppose there is a wind field with a certain direction and strength. Within this wind field, a net is stretched out that hardly obstructs the airflow. If such a net is stretched out in a windy area, we can observe that an object carried by the wind will stick to the net. The force of the wind is necessary to keep the object attached to the net. This force is due to the kinetic energy of air particles; when air particles collide with the object, the kinetic energy of the air particles is transferred to the object, causing it to stick to the net. At this time, the kinetic energy of the air particles is lost, and the overall energy of the wind field also decreases. This decreased energy is then added to the object, but since the object is stuck, from a macroscopic perspective, it appears to be at rest. Since the object is at rest, there is no change in its kinetic energy or its potential energy in the wind field. If we assume that no energy is needed when a force is applied

to an object if the object does not move, then we cannot explain the decrease in the energy of the wind field. In other words, the law of conservation of energy does not hold in this case. Physicists would not be able to tolerate admitting that the law of conservation of energy does not hold, so they would explain that the energy of the wind field was converted into the internal elastic energy or thermal energy of the stationary object. However, if we accept this, then we would have to admit, conversely, that when a force is applied to an object, energy is needed whether the object moves macroscopically or not. This means that we can no longer claim that no energy is needed to hold and support an object in a gravitational field.

8.9 Composition of forces

The resultant force of two or more forces can be calculated using the principle of superposition; it is believed that the individual forces should be added together vectorically. That is,

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \ldots + \mathbf{F}_n \qquad (8.21)$$

This is how it is expressed. Using this idea, if we find the resultant force of two identical forces in opposite directions,

$$\mathbf{F} = \mathbf{F}_1 - \mathbf{F}_1 = 0$$

Therefore, the resultant force is zero. This idea implies that when two sumo wrestlers A and B push against each other in the center of the ring with the same force, neither wrestler moves, so their acceleration is zero, and their resultant force is zero. This idea is not wrong, but in terms of understanding force, it is insufficient on its own. For example, if work is calculated as force $\times$ displacement, then since the two wrestlers do not move, the work they do is zero, meaning that the two wrestlers did absolutely no work, that they slacked off, and their efforts were not rewarded.

If sumo wrestler A exerts a force of $\mathbf{F}_1$ and sumo wrestler B exerts a force of $-\mathbf{F}_1$, then the amount of force exerted by each sumo wrestler is:

The amount of force of sumo wrestler A = |F1|

The amount of force of sumo wrestler B = | - F1|

Therefore, the total amount of force exerted by the two sumo wrestlers A and B can be calculated by adding these two amounts together.

$$\text{The total amount of force } = |\mathbf{F}_1| + |-\mathbf{F}_1| = |2\mathbf{F}_1|$$

In general terms,

$$|\mathbf{F}| = |\mathbf{F}_1| + |\mathbf{F}_2| + |\mathbf{F}_3| + ... + |\mathbf{F}_n| \qquad (8.22)$$

That is the case.

If two sumo wrestlers, A and B, were to exert this force for t seconds, then their impulse would be Since it represents energy, the energy released by the two of them is,

$$E = |2\mathbf{F}_1|t = 2F_1 t$$

This proves that the two sumo wrestlers were not being lazy.

As mentioned in Chapter 5 on the conservation of momentum, when dealing with quantities expressed as vectors, it is necessary to consider both their direction and their magnitude. Even if the resultant force of the forces expressed as vectors becomes zero, the resultant force of the forces expressed as magnitudes does not necessarily become zero.

By expressing forces in both forms (8.21) and (8.22), we can mathematically distinguish between the case where a force is applied to an object but the object does not move, and the case where no force is applied to the object at all.